Condition for Explosion

by

Jeremiah W. Murphy

(Florida State University)

An Integral Condition for the Explosion of Core-collapse SNe

by
Jeremiah W. Murphy
(Florida State University)
&
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(LANL)


What is the mechanism of explosion?

Let's assume that the delayed-neutrino mechanism works

What are the conditions for explosion?

Imagine…

An explosion condition that

…is derived analytically (not heuristic)

…identifies how close a simulation is to explosion.

…can predict which of T. Sukhbold's progenitors will explode.

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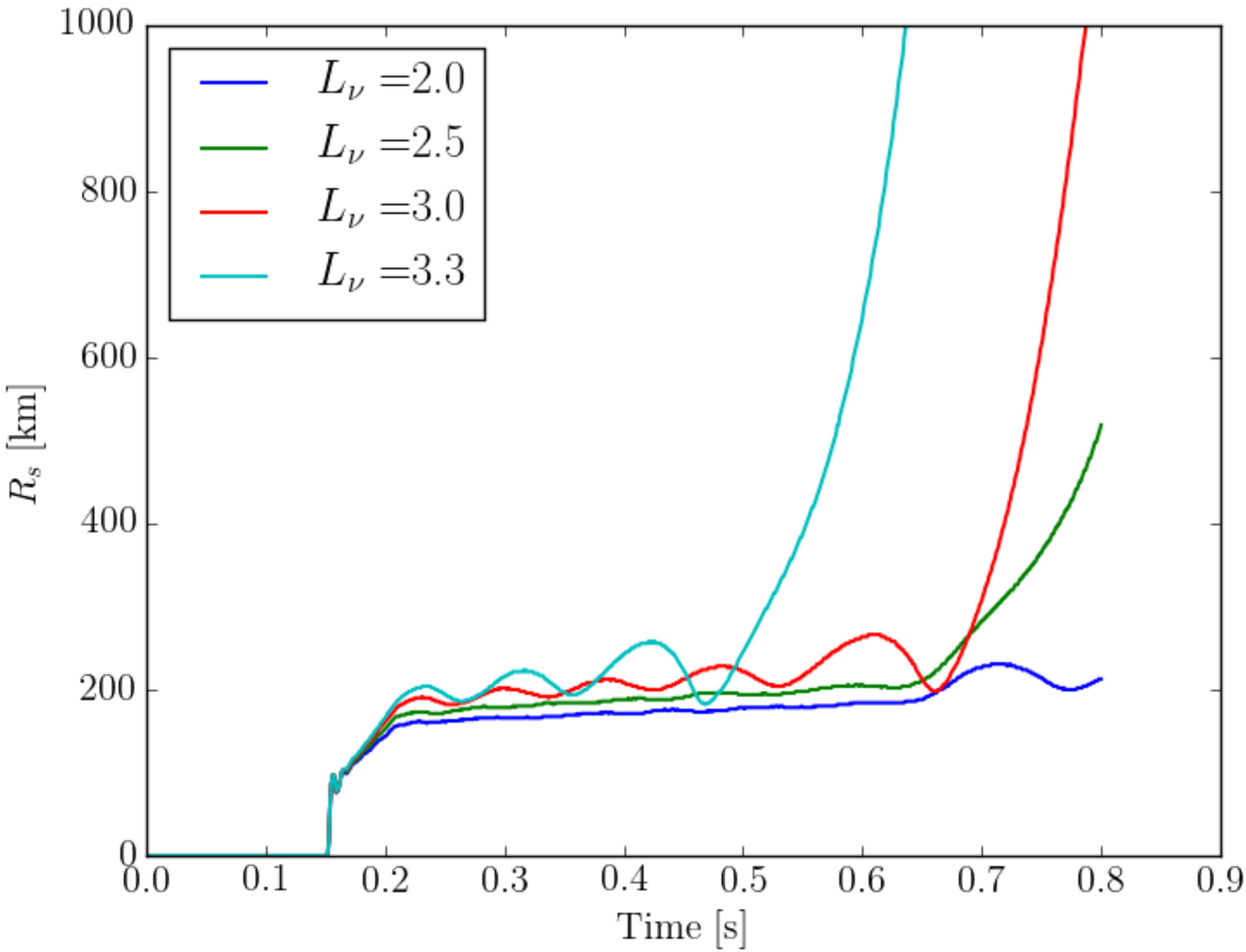
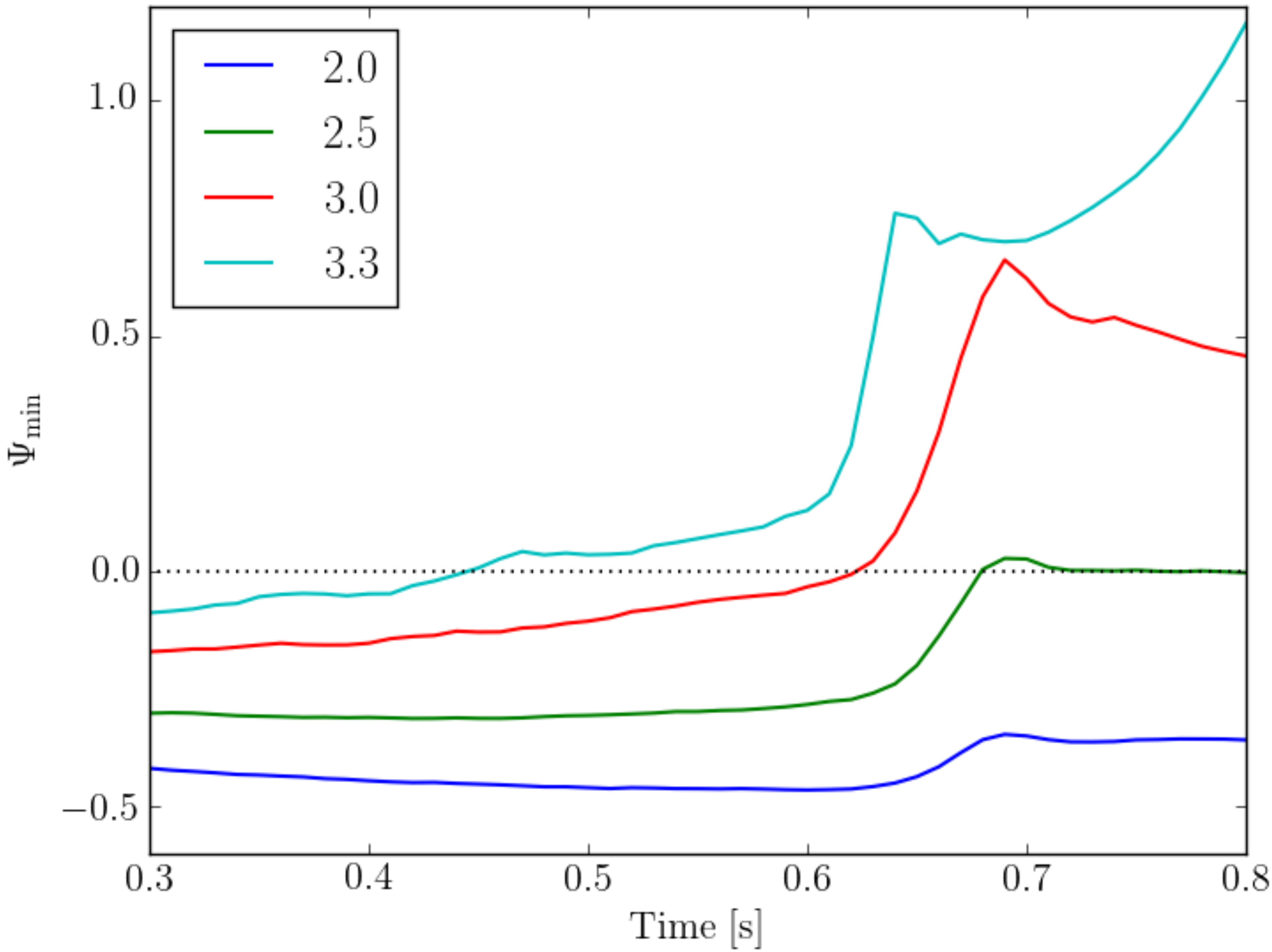
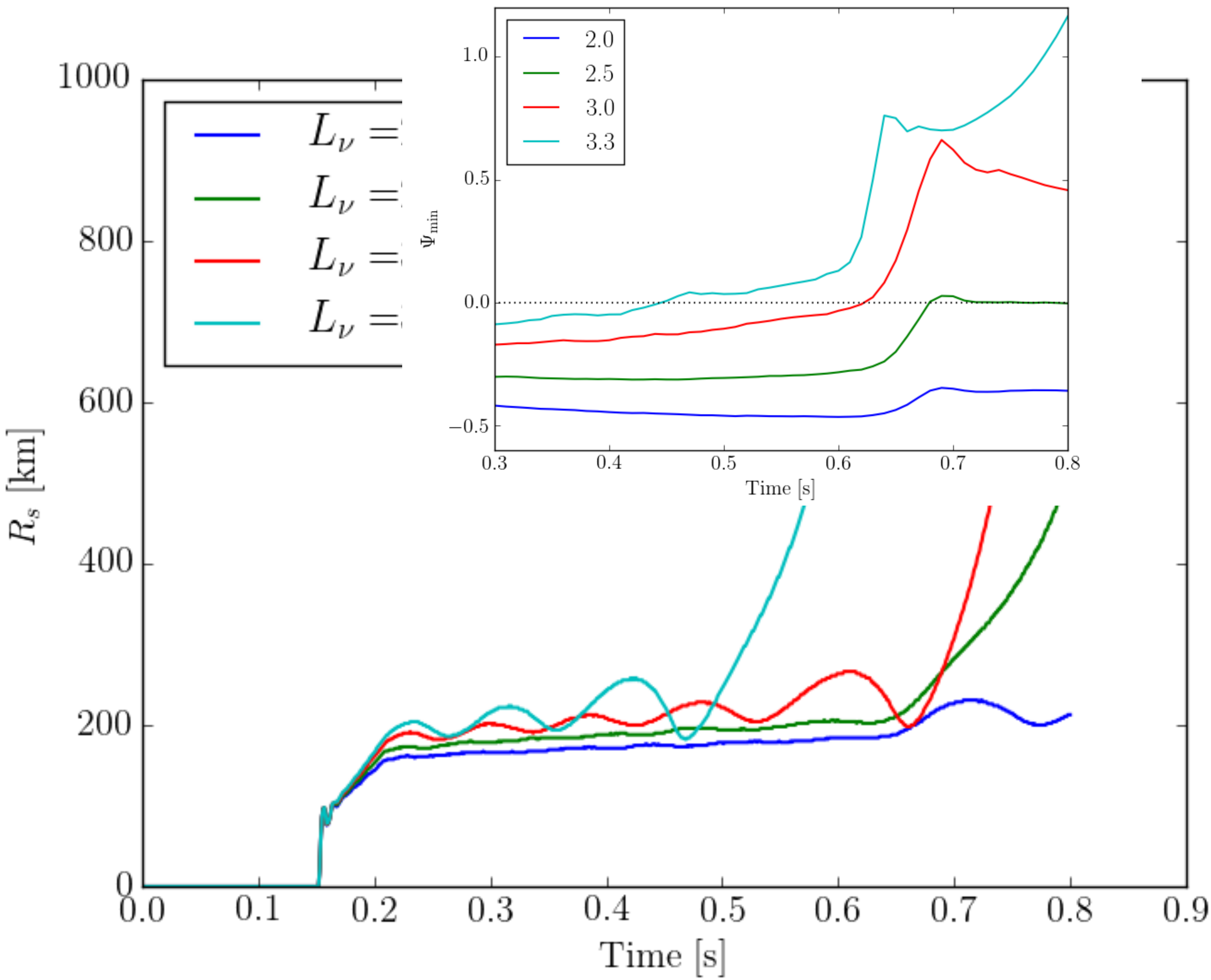


L_ν =2.0
L_ν =2.5
L_ν =3.0
L_ν =3.3
1000
800
600
400
200
0
R_s [km]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Time [s]



2.0
2.5
3.0
3.3
1.0
0.5
0.0
−0.5
$\Psi_{\min}$
0.3
0.4
0.5
0.6
0.7
0.8
Time [s]

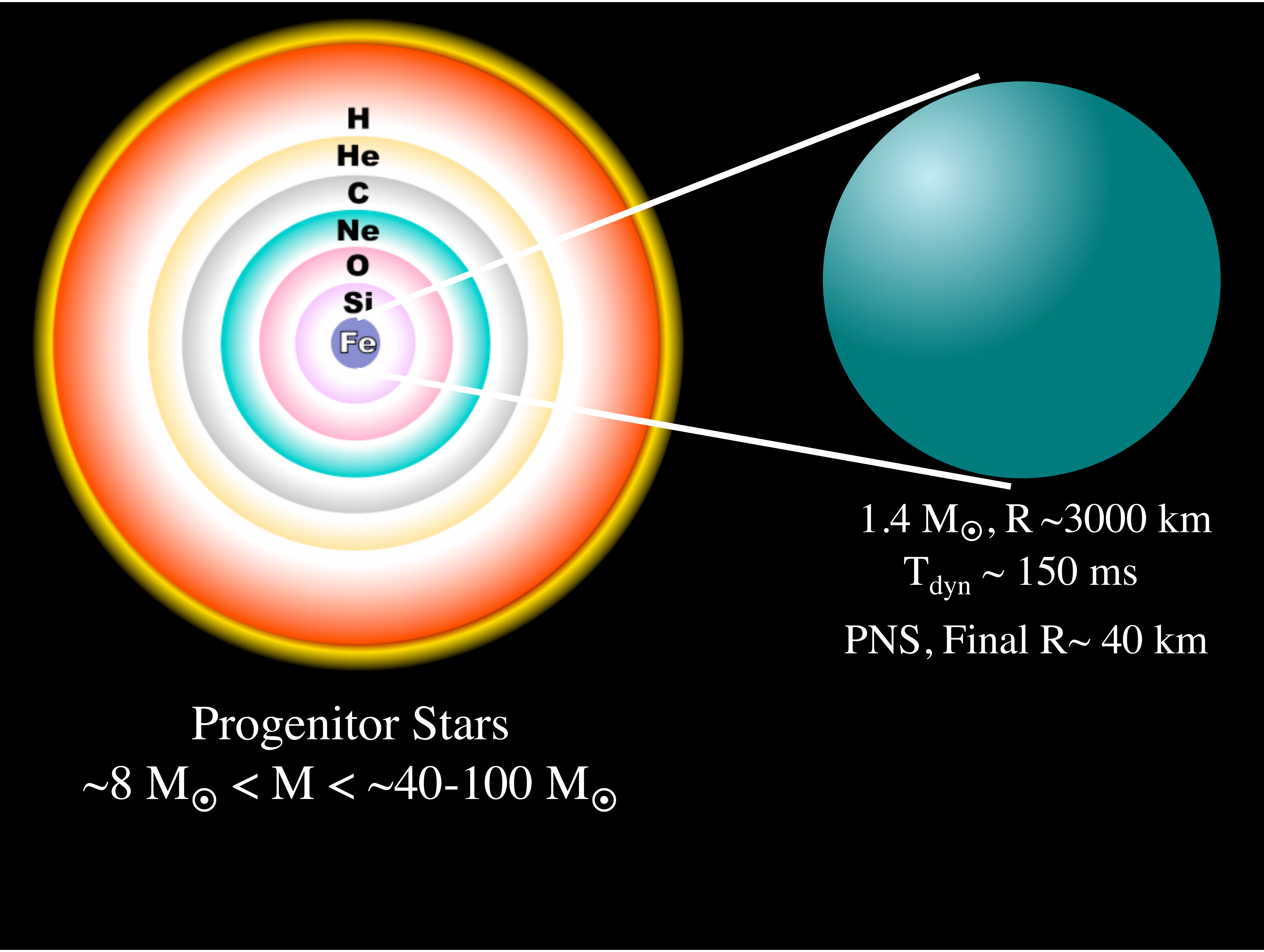


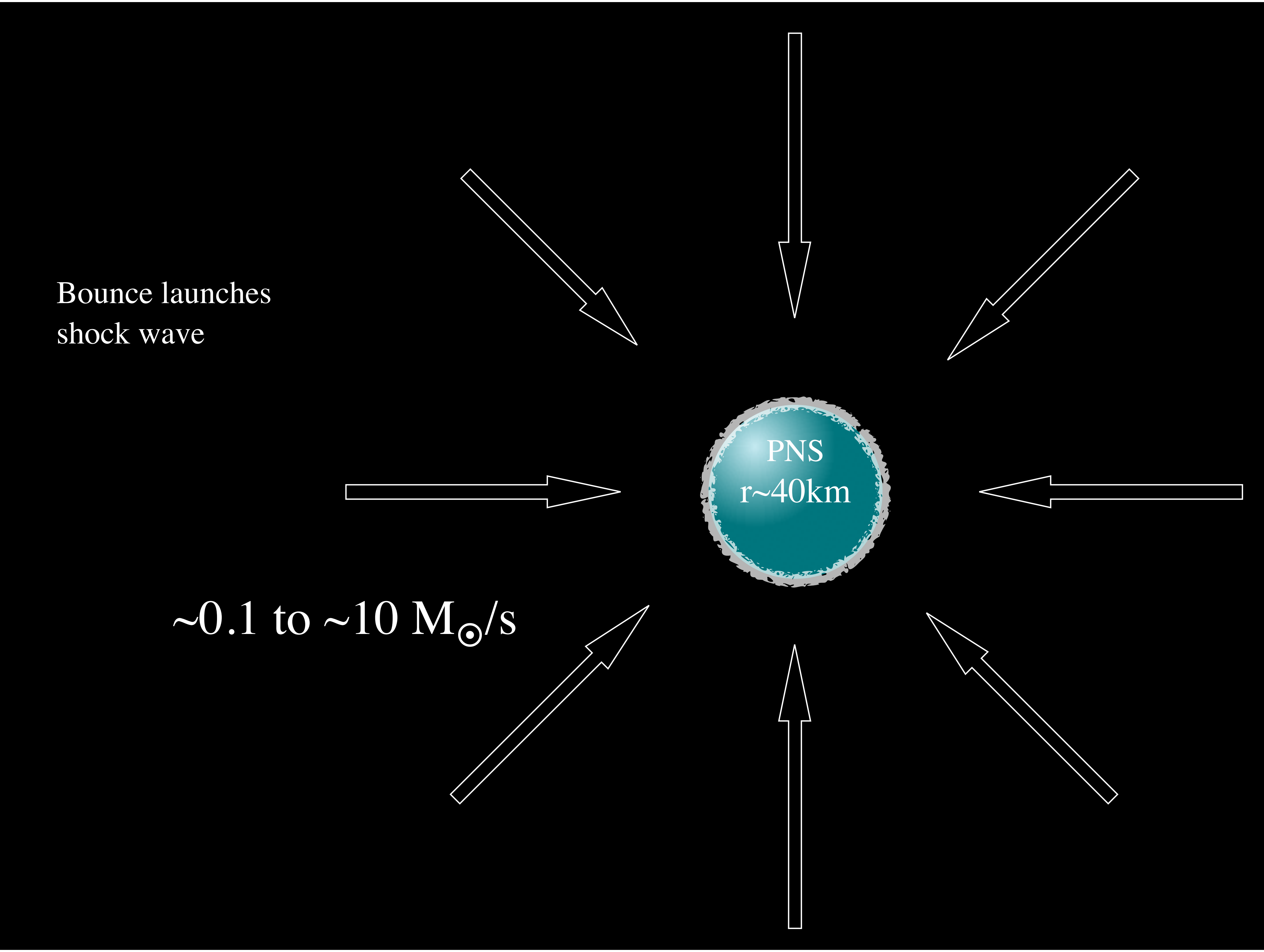
$L_\nu =$
$L_\nu =$
$L_\nu =$
$L_\nu =$
1000
800
600
400
200
0
R_s [km]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Time [s]
2.0
2.5
3.0
3.3
1.0
0.5
0.0
−0.5
$\Psi_{\min}$
0.3
0.4
0.5
0.6
0.7
0.8
Time [s]

So what is Ψ_{min}?

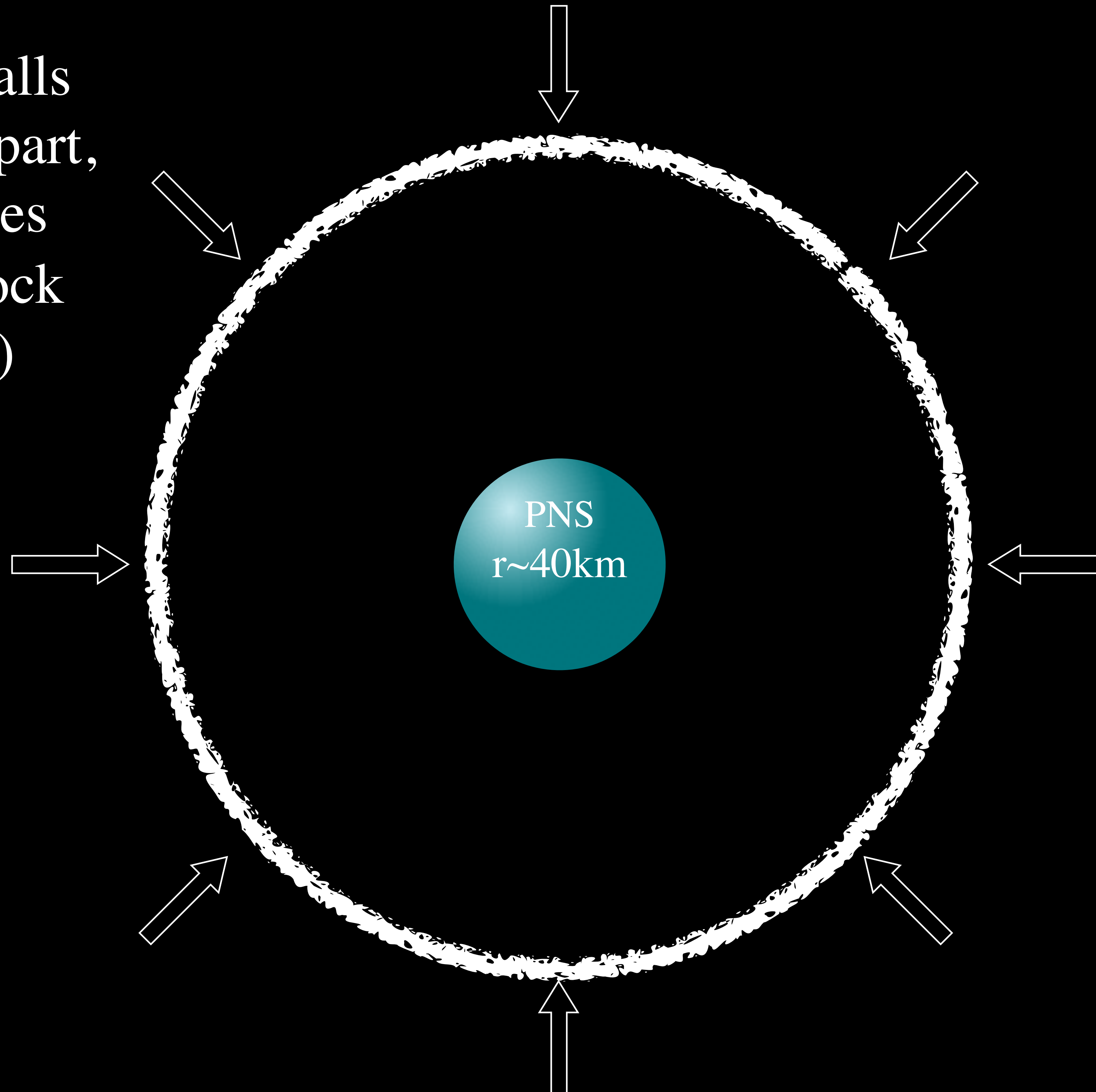
So what is Ψ_{min}?

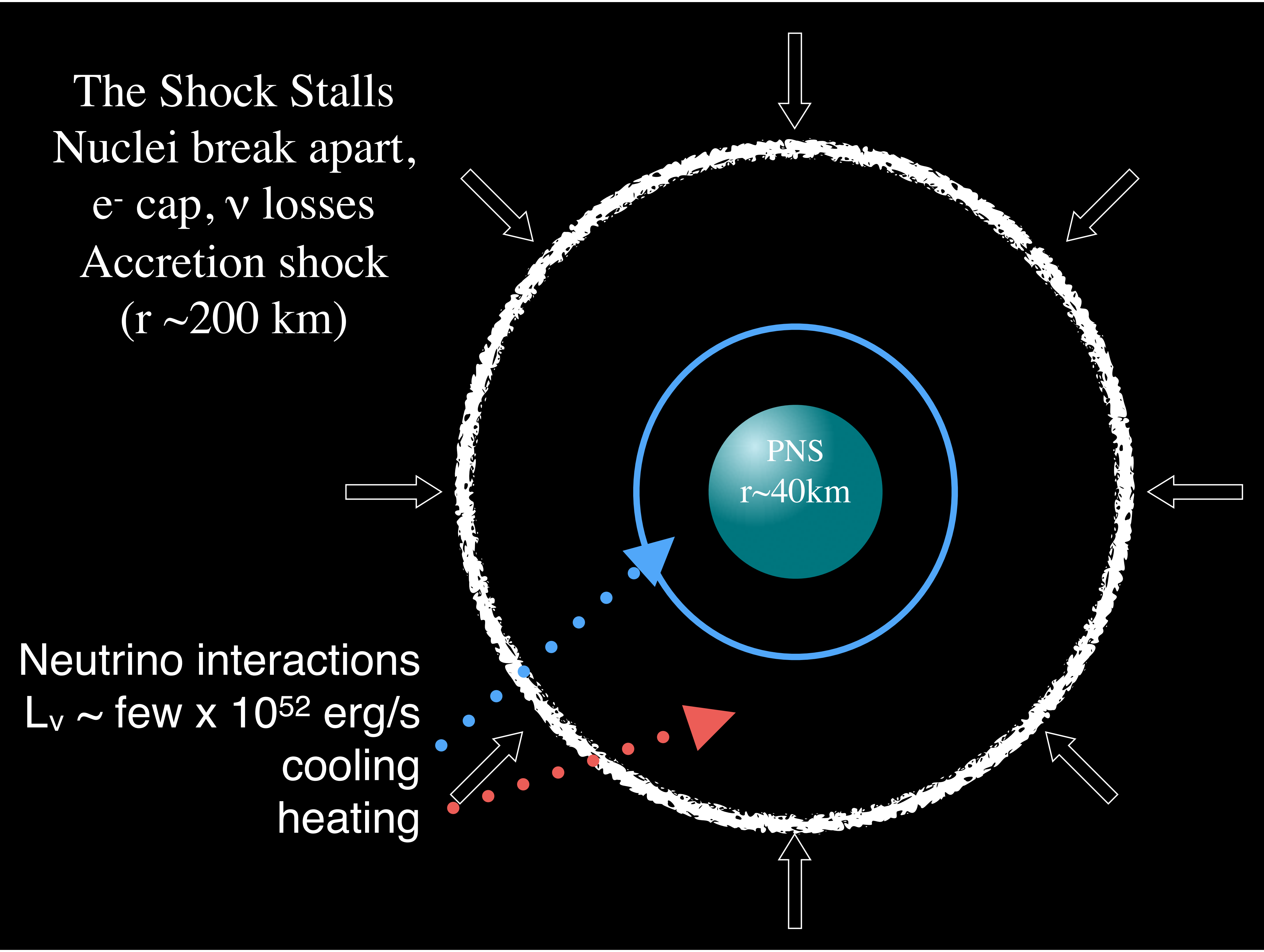
Before I derive this, let me set the stage


H
He
C
Ne
O
Si
Fe
1.4 $M_\odot$, R ~3000 km
T_{dyn} ~ 150 ms
PNS, Final R~ 40 km
Progenitor Stars
~8 $M_\odot$ < M < ~40-100 $M_\odot$


Bounce launches
shock wave
~0.1 to ~10 $M_\odot$/s
PNS
r~40km

The Shock Stalls
Nuclei break apart,
e^- cap, ν losses
Accretion shock
(r ~200 km)




The Shock Stalls
Nuclei break apart,
e^- cap, ν losses
Accretion shock
(r ~200 km)
PNS
r~40km
Neutrino interactions
L_ν ~ few x 10^{52} erg/s
cooling
heating

Important Parameters:

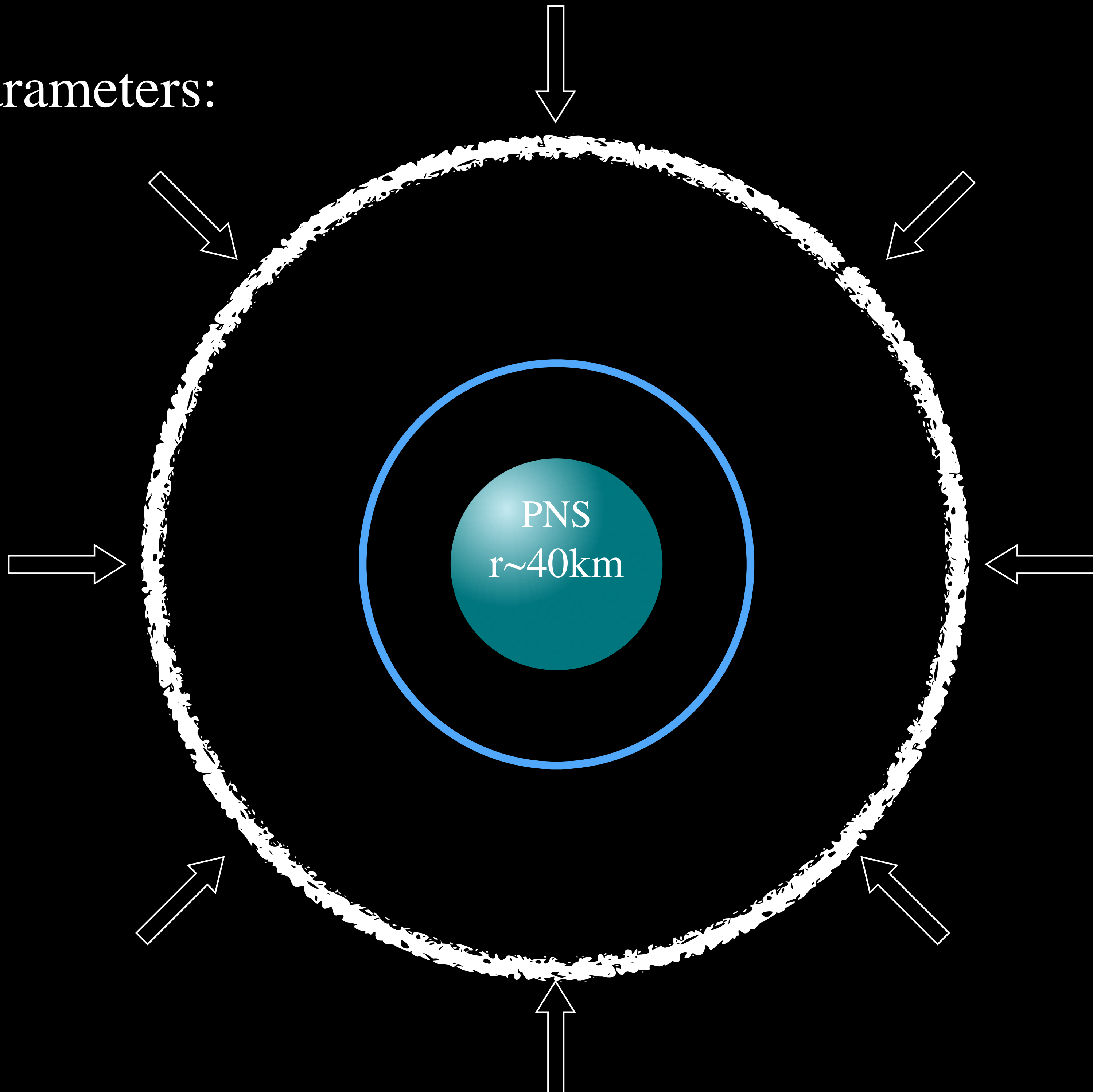
L_ν

$\dot{M}$

T_ν

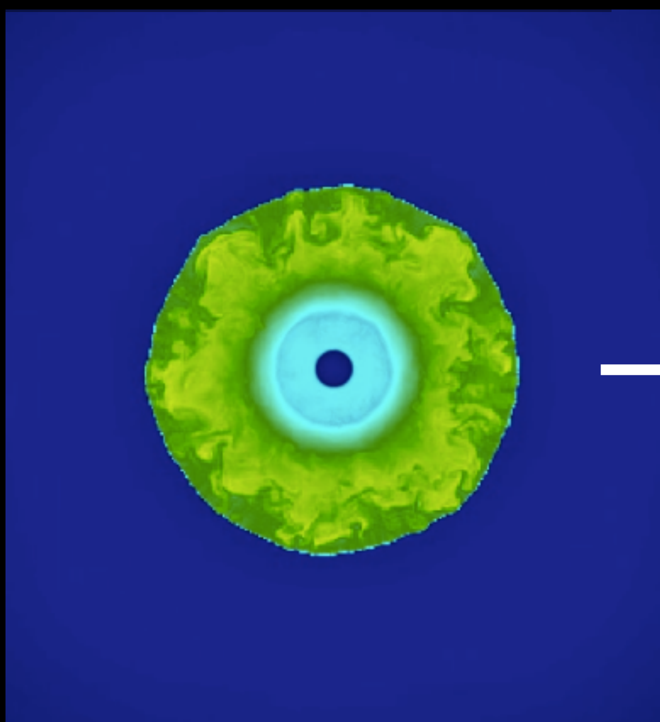
R_{NS}

M_{NS}



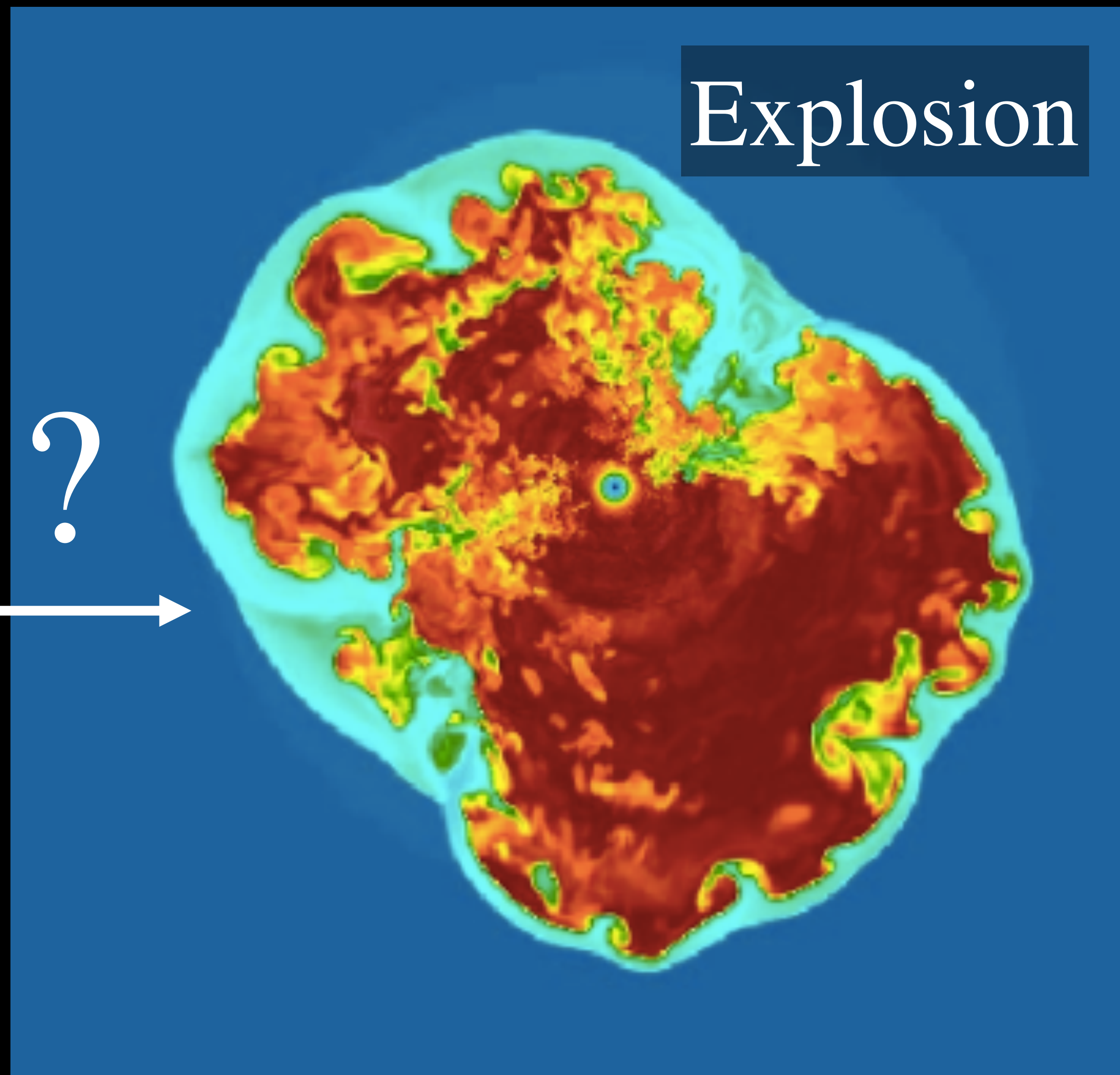
Fundamental Question of Core-Collapse Theory

Stalled Shock



Murphy et al. 2013

?

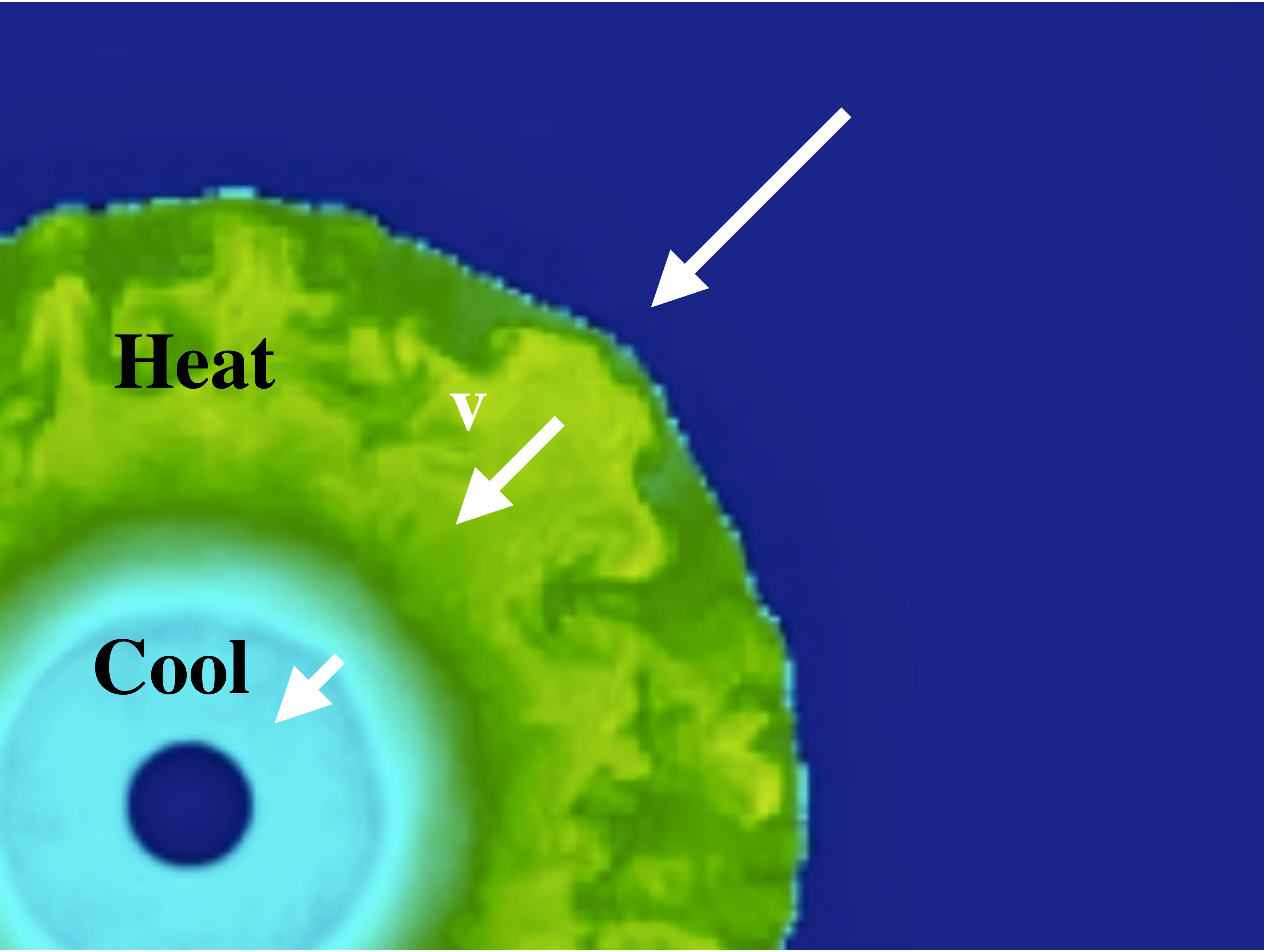


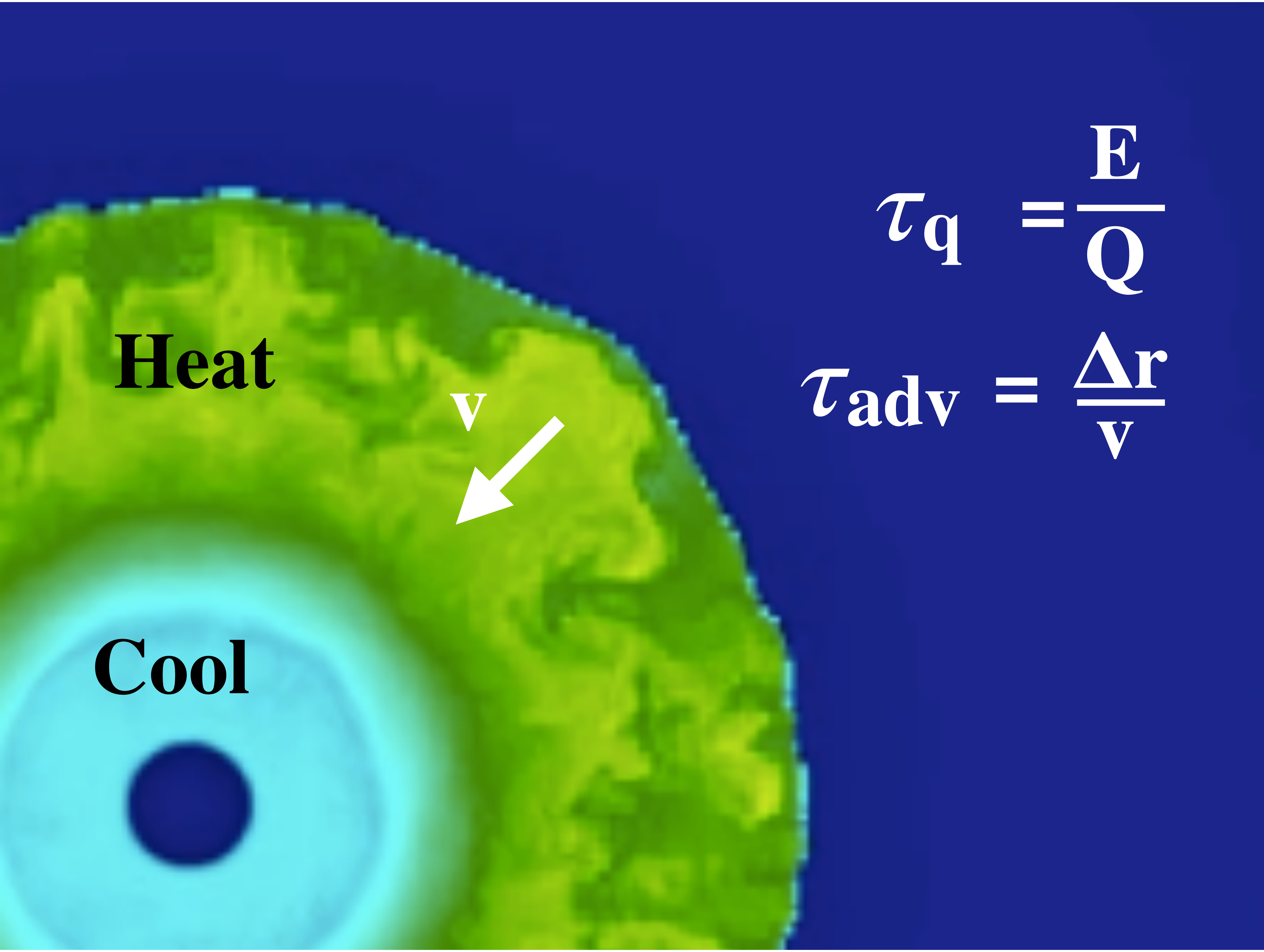
What are the conditions for shock revival?

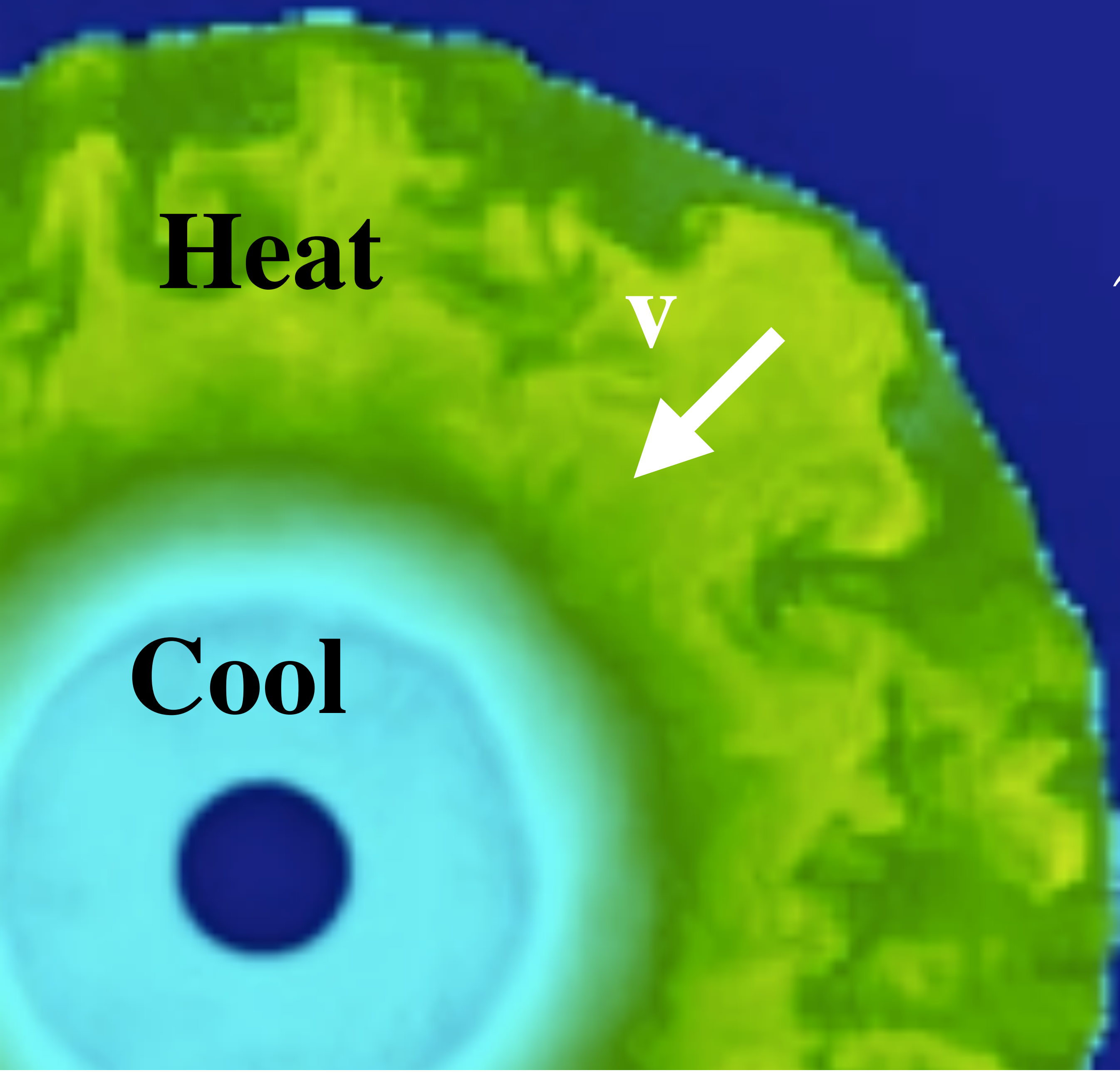
$$v_s \geq 0$$

A couple of examples

$$v_s \geq 0$$


Heat
v
Cool


Heat
Cool
v
$\tau_q = \frac{E}{Q}$
$\tau_{adv} = \frac{\Delta r}{v}$

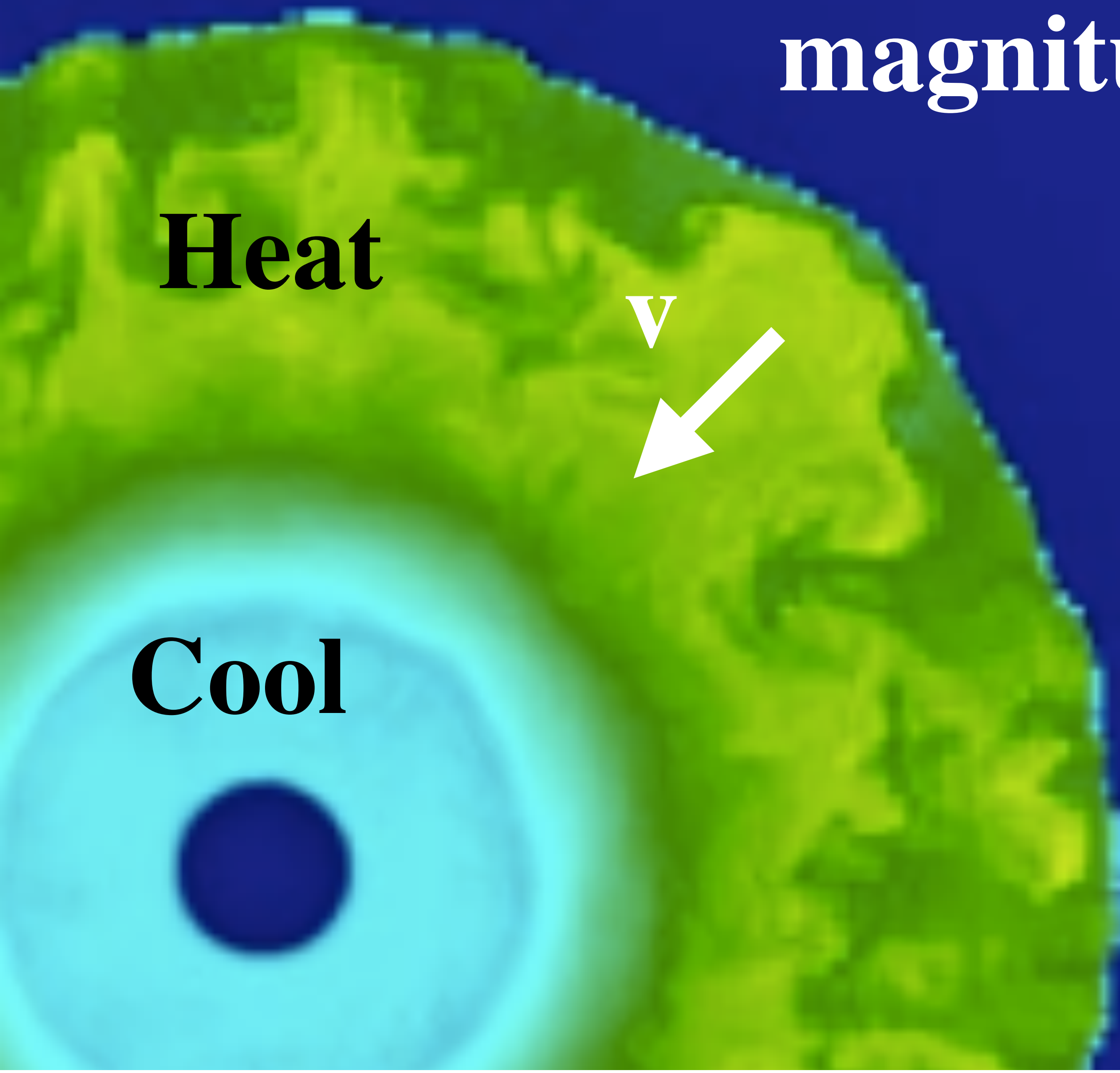


$$\tau_q = \frac{E}{Q}$$

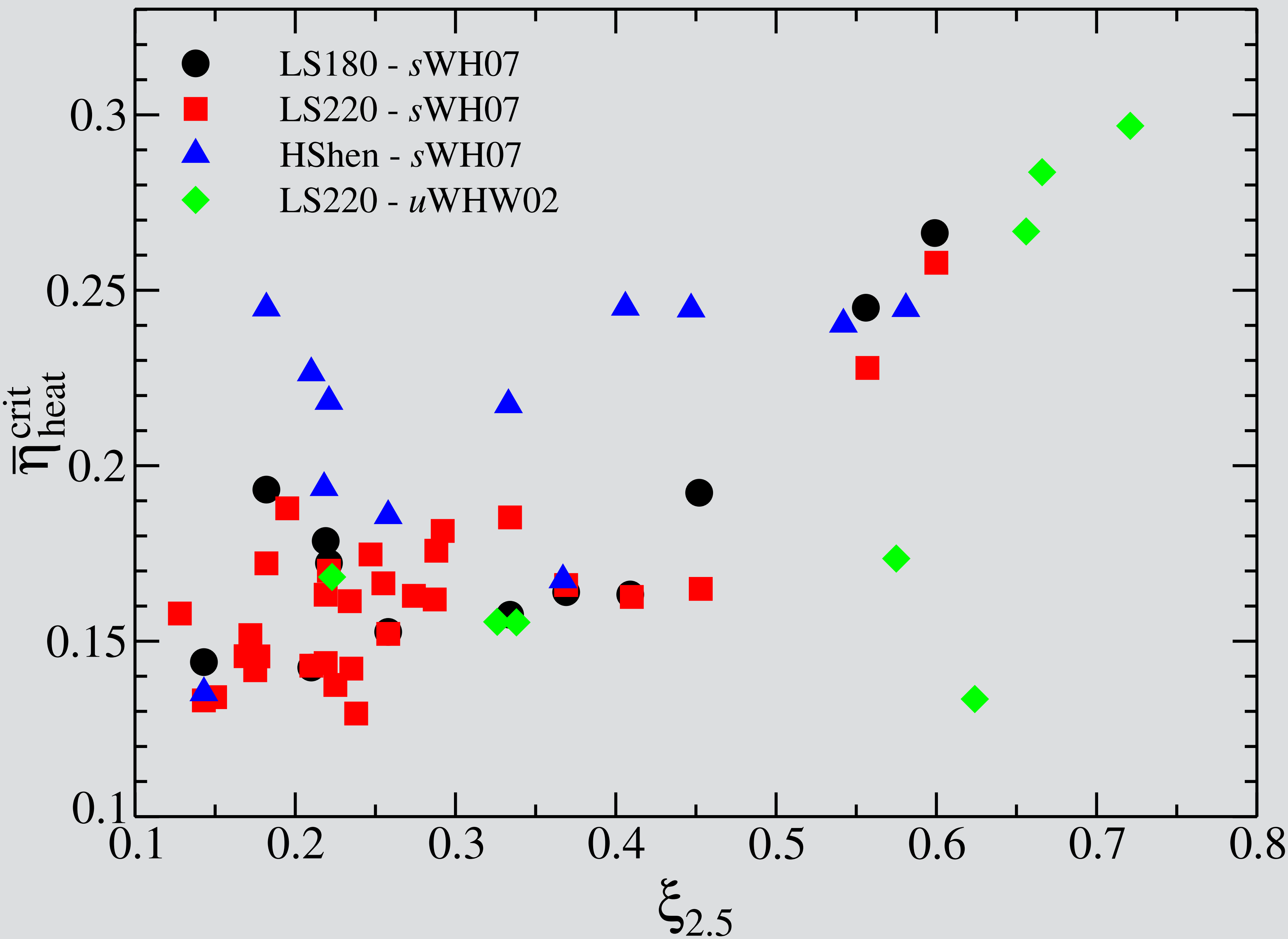
$$\tau_{adv} = \frac{\Delta r}{v}$$

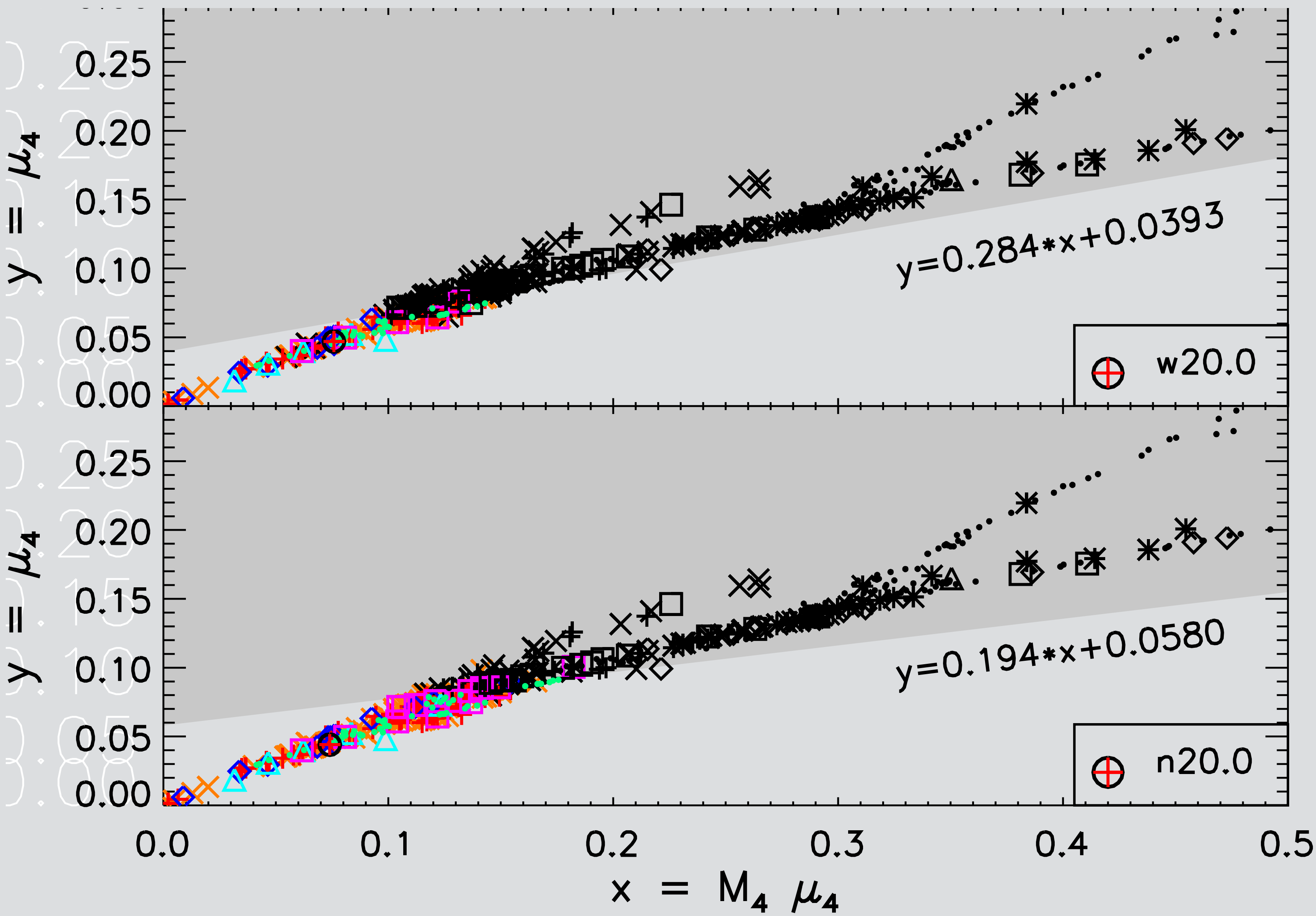
$$\frac{\tau_{adv}}{\tau_q} \gtrsim 1$$

But this is order of magnitude



$$\frac{\tau_{adv}}{\tau_q} \gtrsim 1$$


LS180 - sWH07
LS220 - sWH07
HShen - sWH07
LS220 - uWHW02
$\bar{\eta}_{\rm heat}^{\rm crit}$
$\xi_{2.5}$
0.1
0.15
0.2
0.25
0.3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8



Ertl et al. 2015

Important Parameters:

L_ν

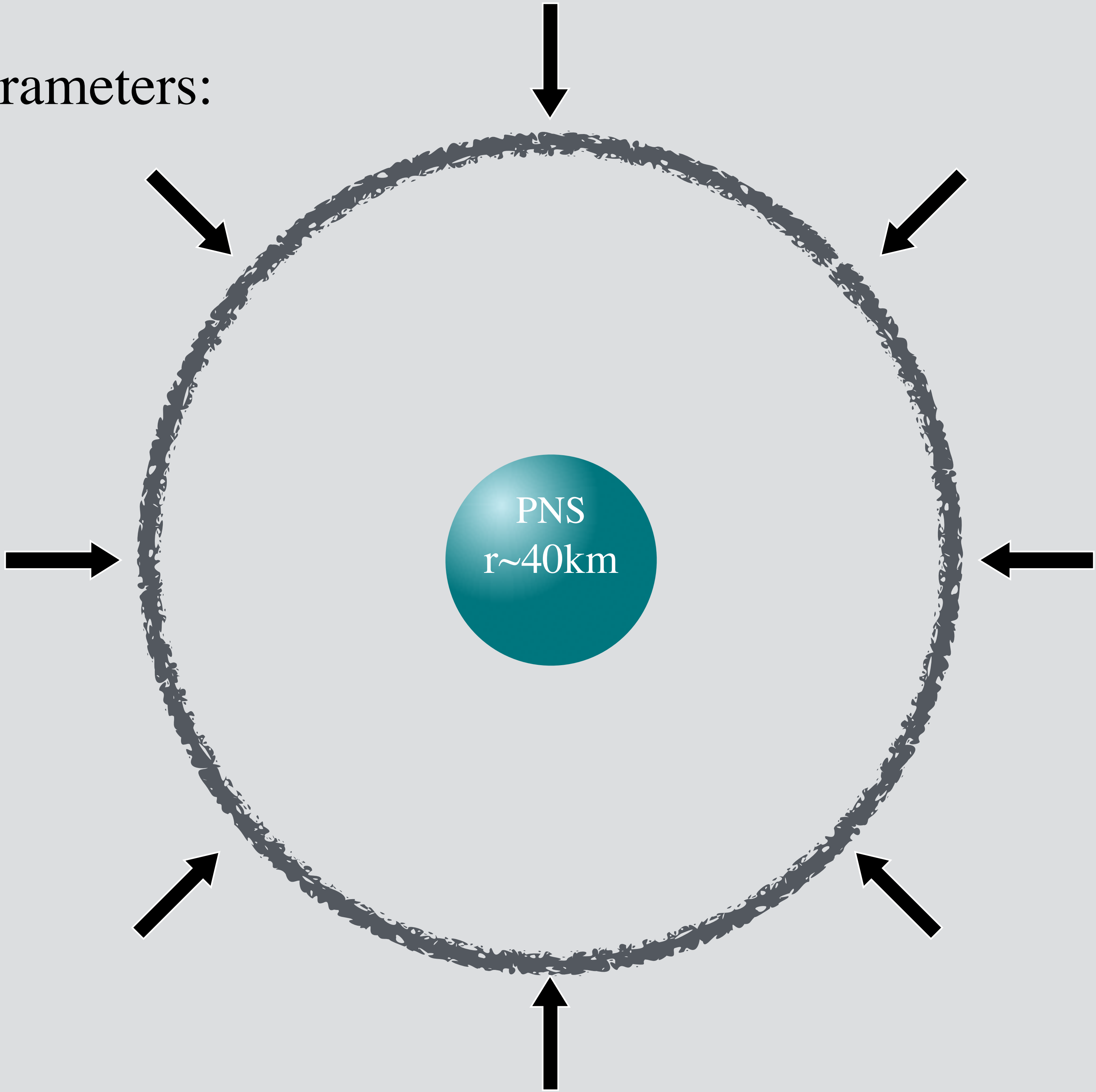
$\dot{M}$

T_ν

R_{NS}

M_{NS}

$$\frac{\partial(.)}{\partial t} = 0$$



Important Parameters:

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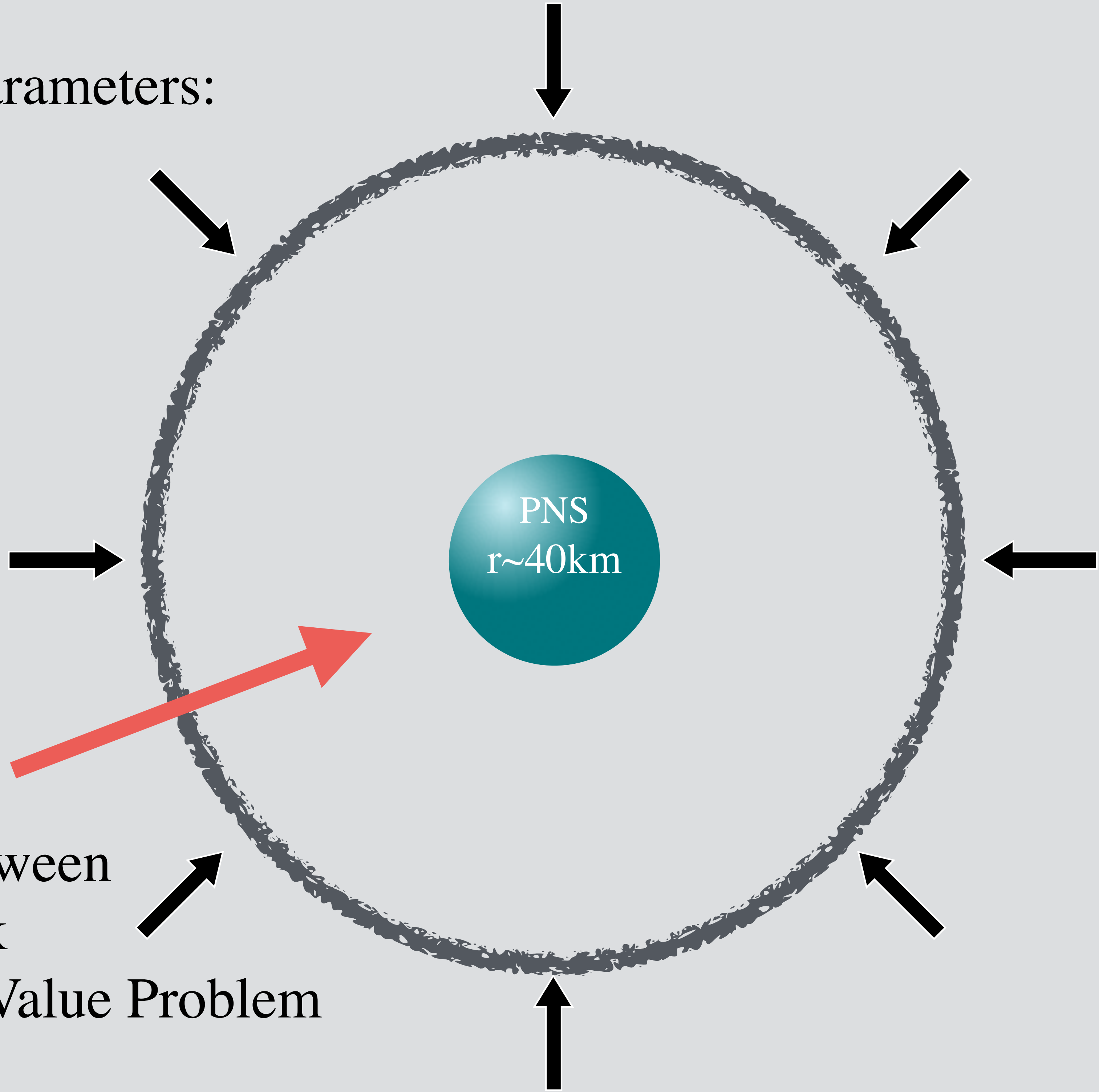
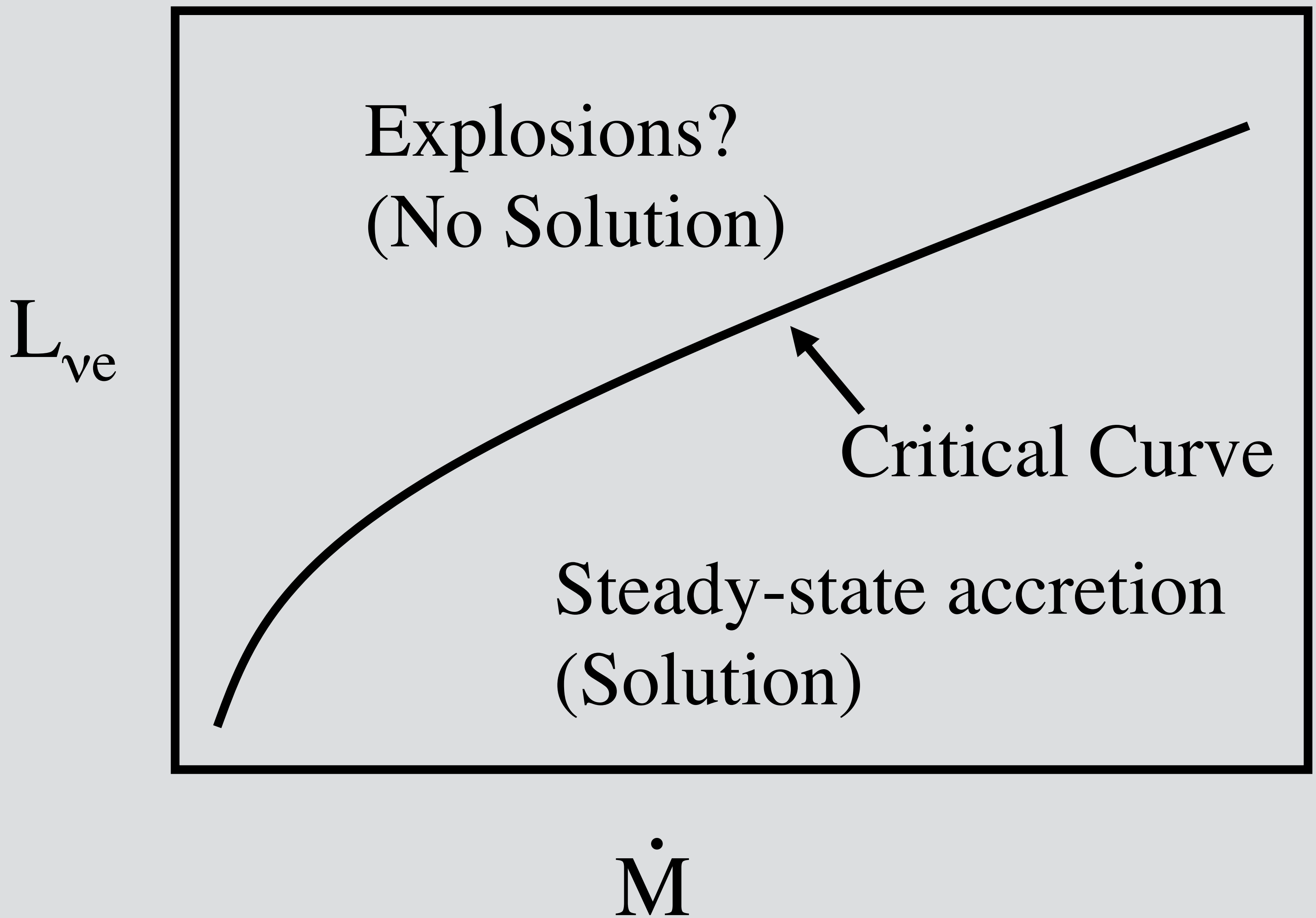
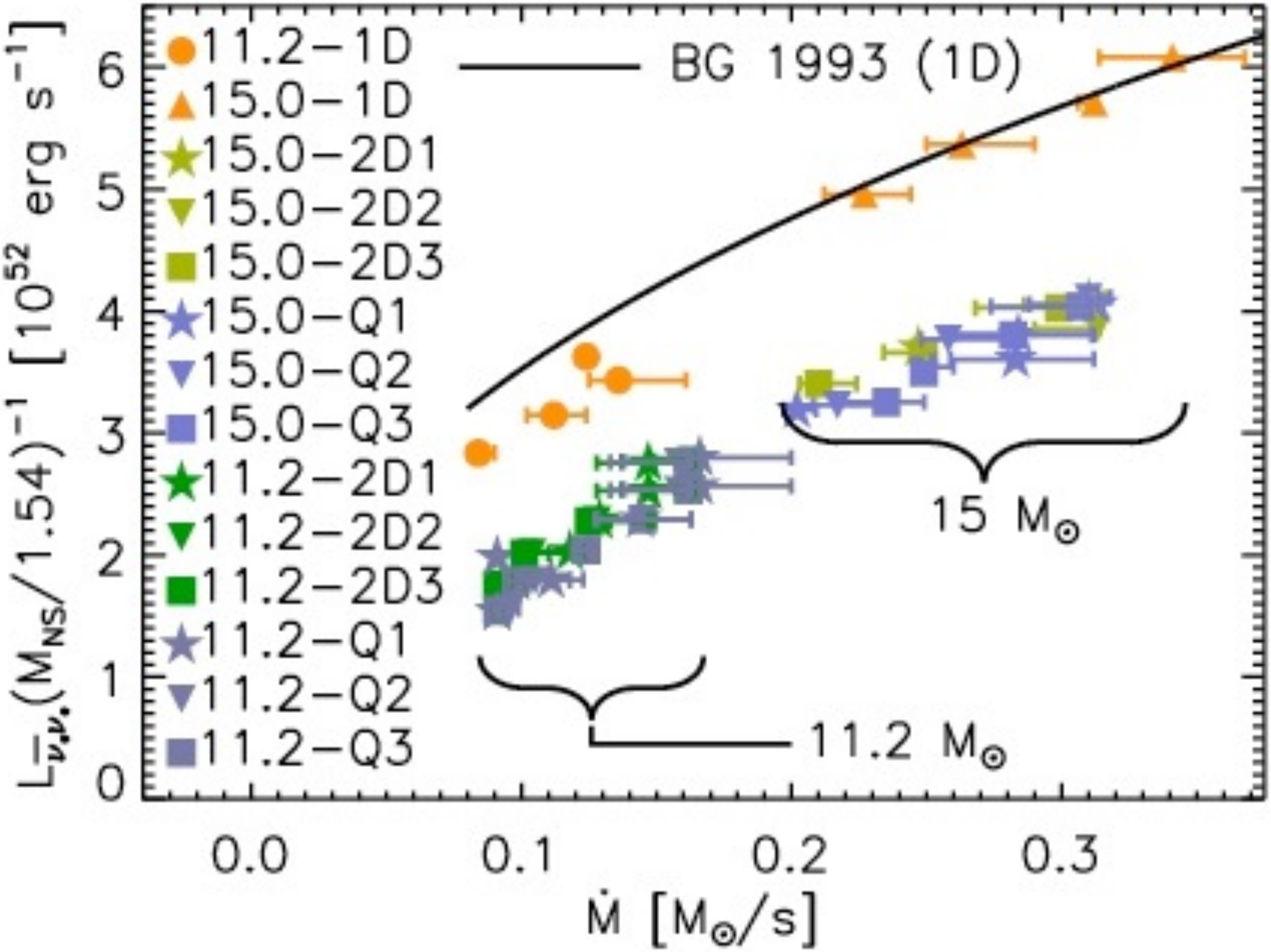


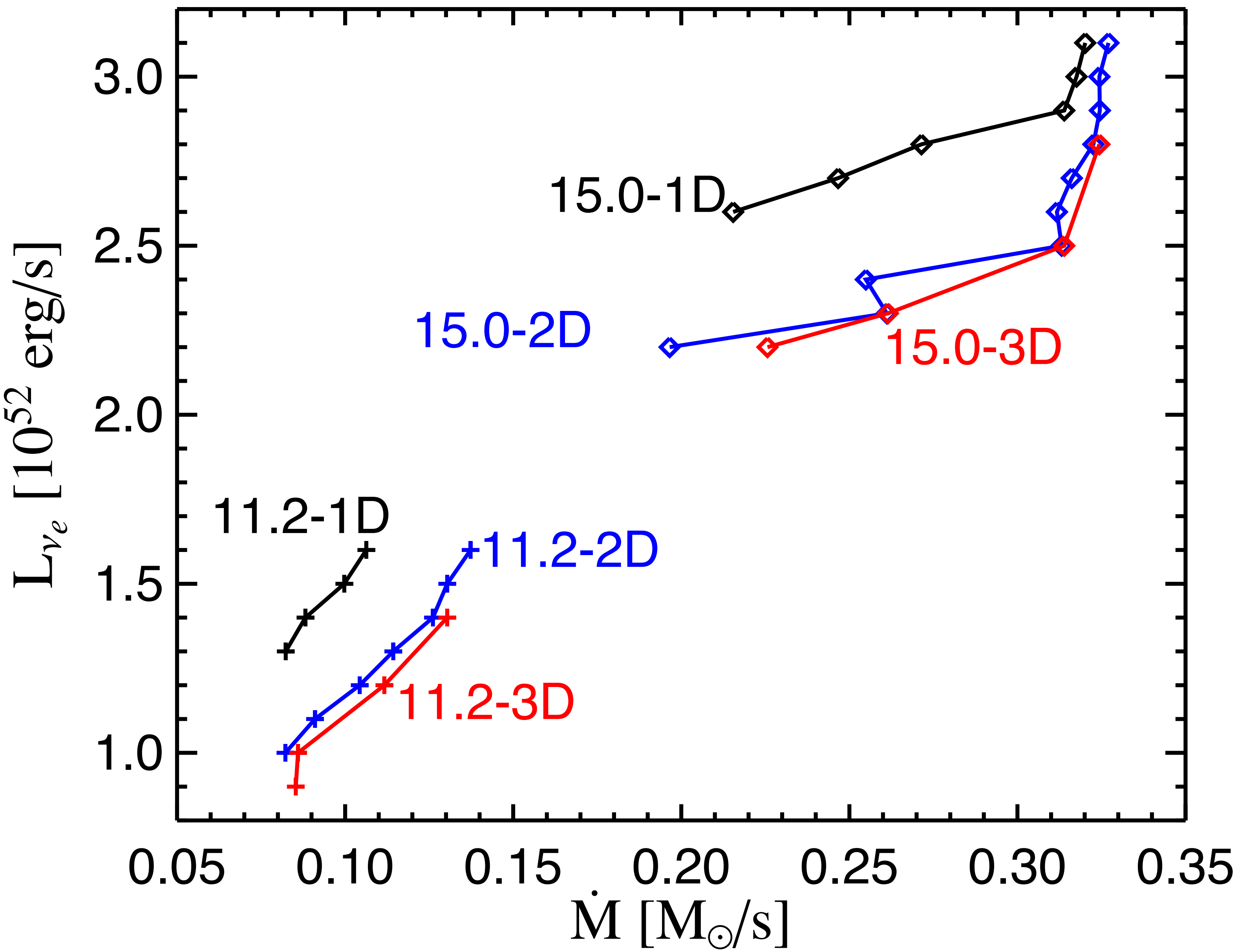
Steady-state
Solutions between
NS and shock
A Boundary Value Problem

Burrows & Goshy '93
Steady-state solution (ODE)

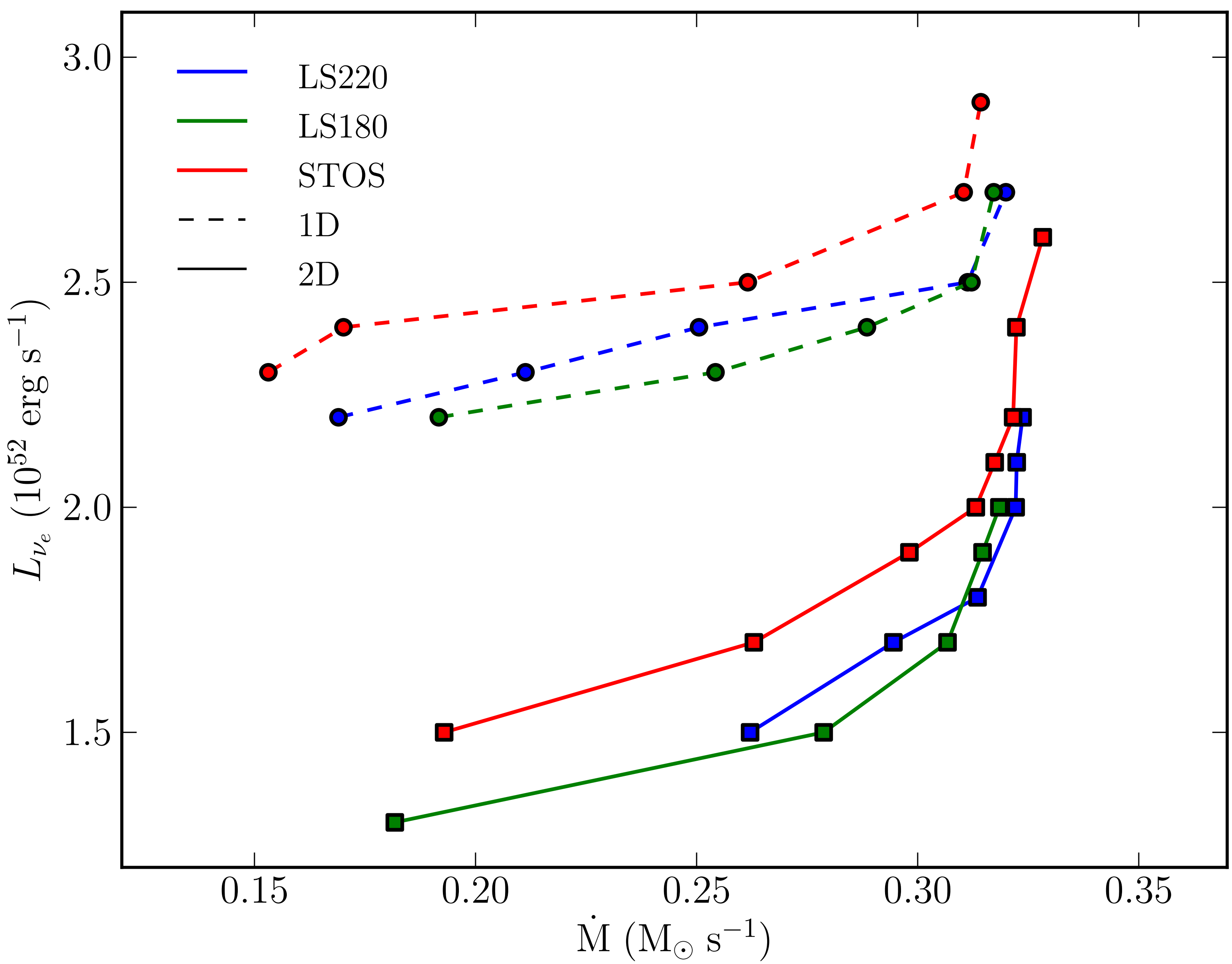


Murphy & Burrows '08





Hanke et al 2011



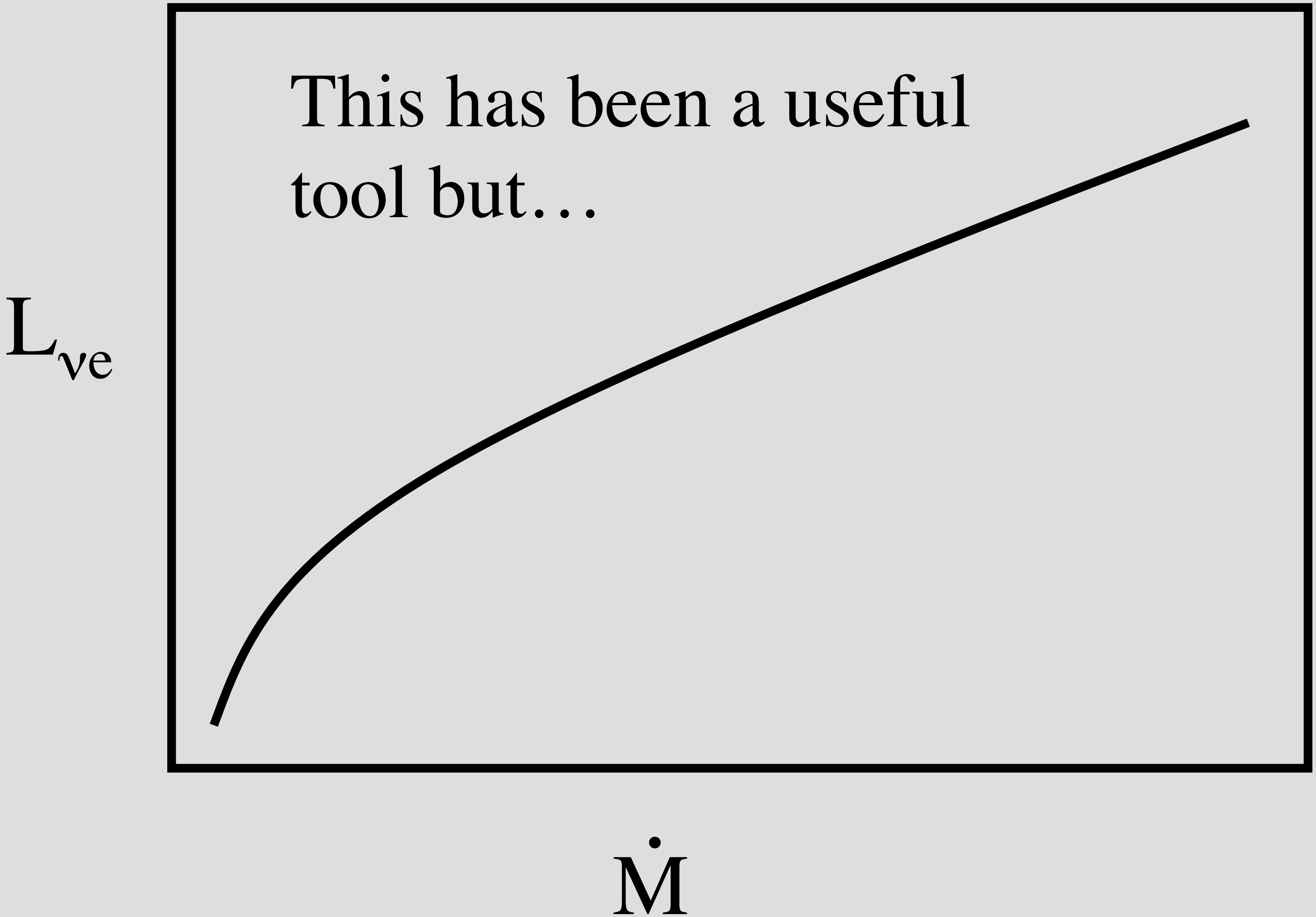
LS220
LS180
STOS
1D
2D
3.0
2.5
2.0
1.5
0.15
0.20
0.25
0.30
0.35
L_{ν_e} (10^{52} erg s^{-1})
$\dot{M}$ ($M_\odot$ s^{-1})

Couch 2012

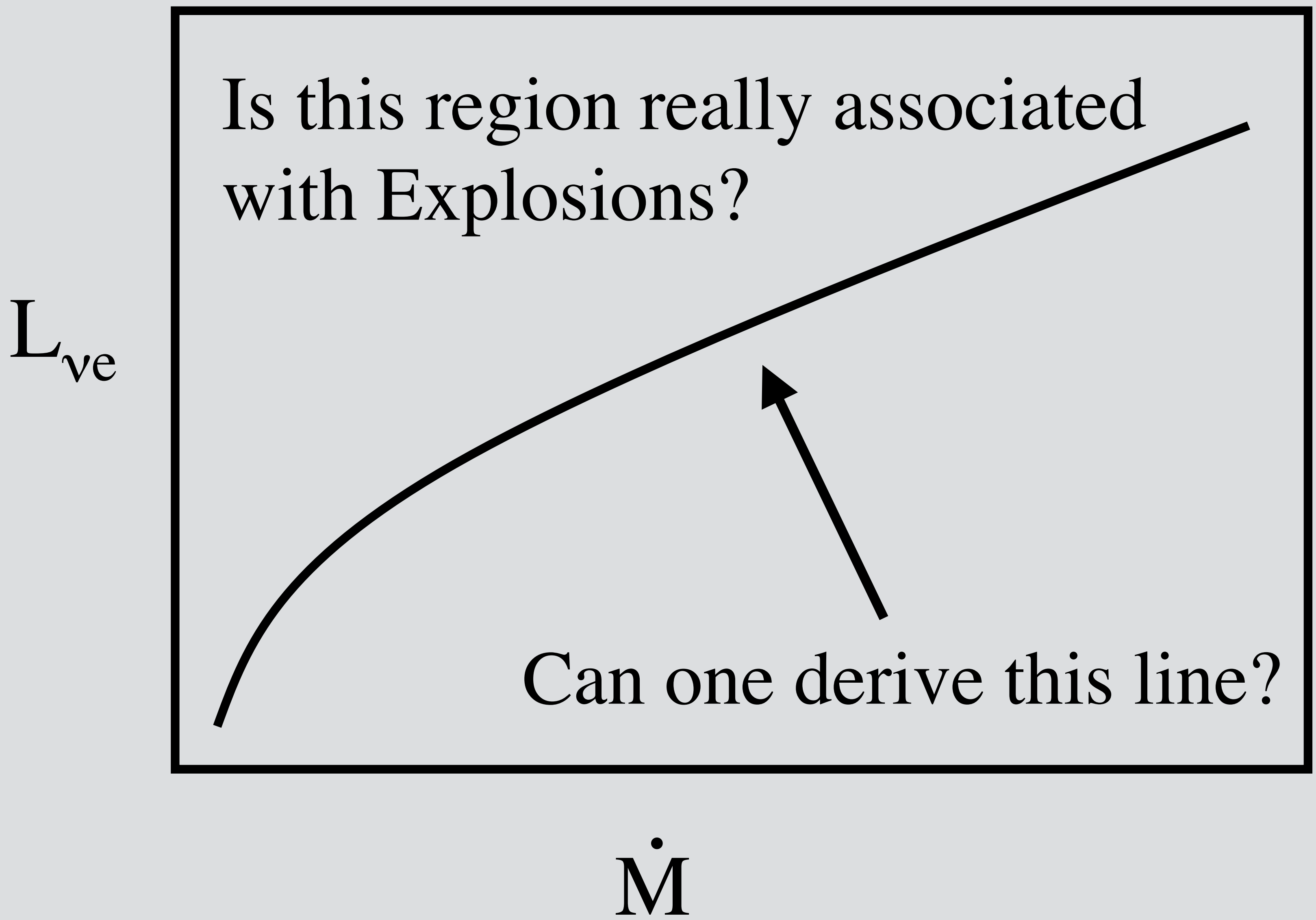
2D & 3D critical luminosity lower than 1D

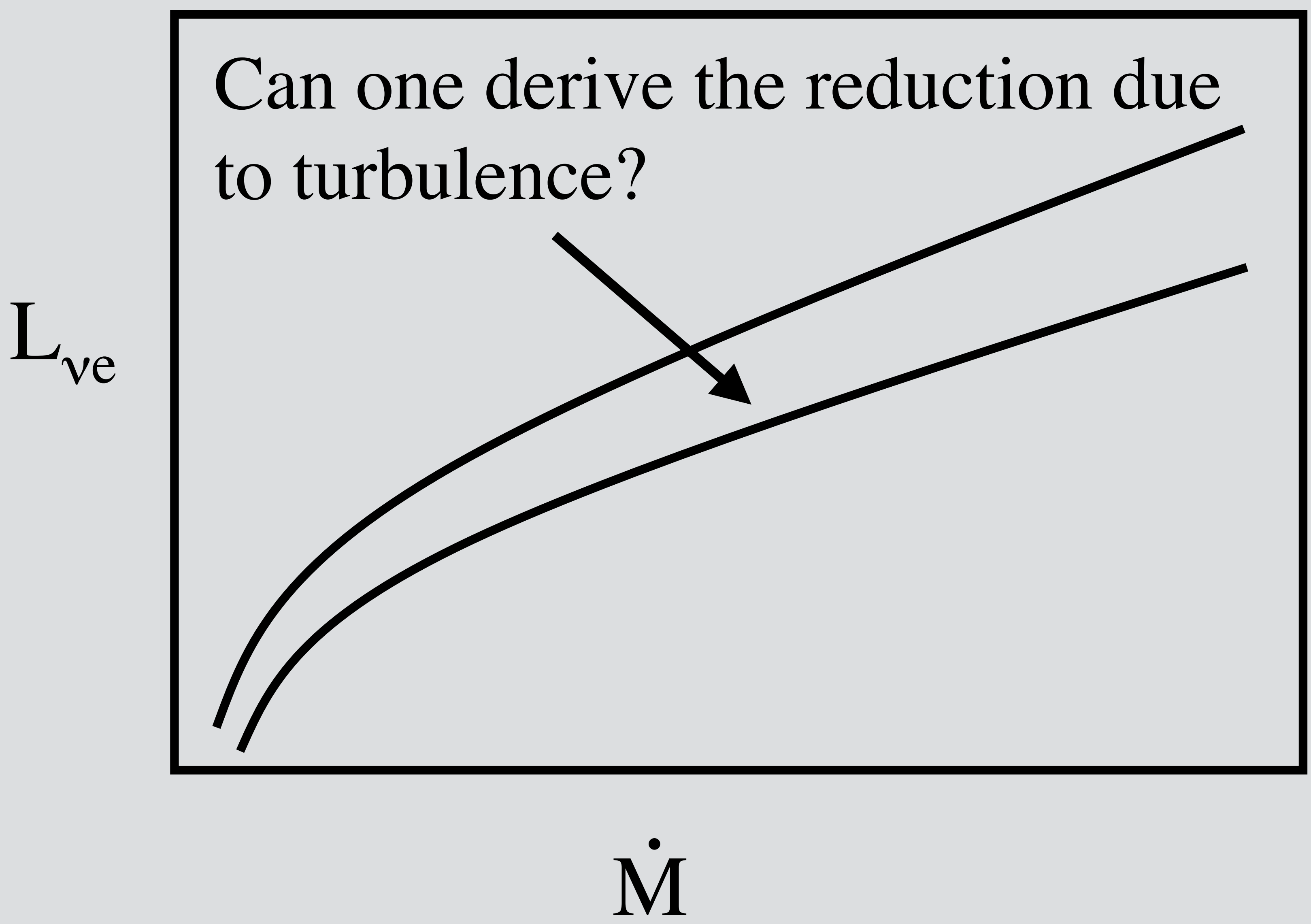
Turbulence plays an important role!

Murphy & Burrows 2008
Murphy & Meakin 2012
Burrows, Dolence, and Murphy 2012
Murphy, Burrows, and Dolence 2013
Dolence, Burrows, and Murphy 2013



This has been a useful
tool but…
Lνe
Ṁ


Is this region really associated with Explosions?
$L_{\nu e}$
Can one derive this line?
$\dot{M}$



Can one derive the reduction due to turbulence?
$L_{\nu e}$
$\dot{M}$

Yes

Let's start with two assumptions:

1. $v_s \geq 0$ is the condition for explosion
2. Integral condition will be illuminating

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2. Integral condition will be illuminating

$$\frac{d^2x}{dt^2} = \frac{f}{m} \quad \text{or} \quad \frac{1}{2}v^2 + \phi = \text{const.}$$

Let's start with two assumptions:

1. $v_s \geq 0$ is the condition for explosion
2. Integral condition will be illuminating

$$\frac{d^2x}{dt^2} = \frac{f}{m} \quad \text{or} \quad \frac{1}{2}v^2 + \phi = \text{const.}$$

Will use $v_s \geq 0$ to derive an integral condition for explosion.

Governing Conservation equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot F = S$$

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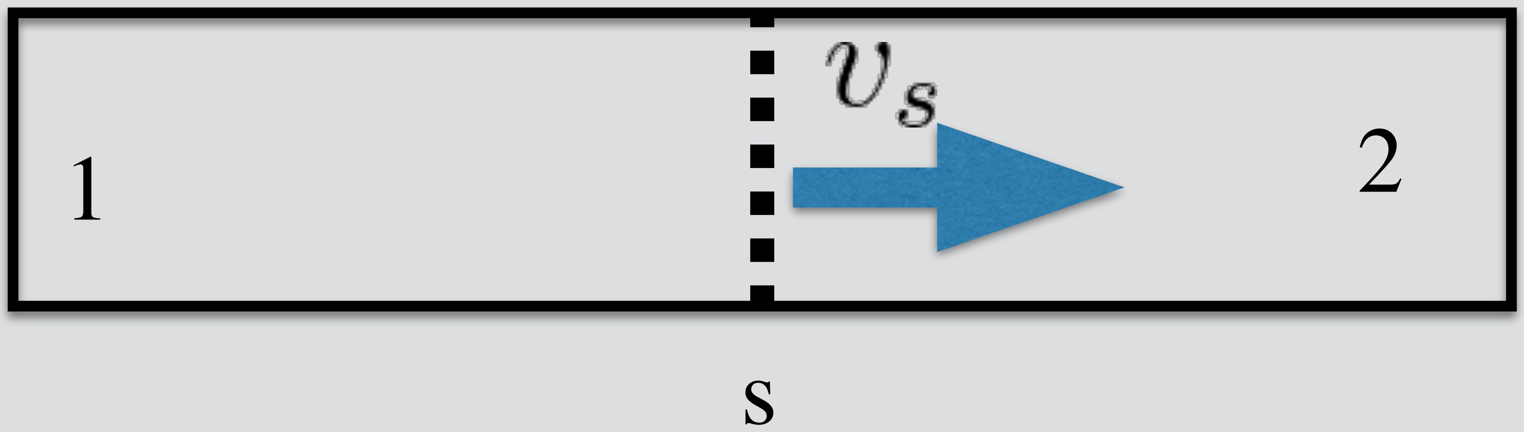
$$\frac{\partial \rho}{\partial t} + \nabla \cdot F = S$$

Change of mass in box $= A_1 F_1 - A_2 F_2 + \int S \, dV$

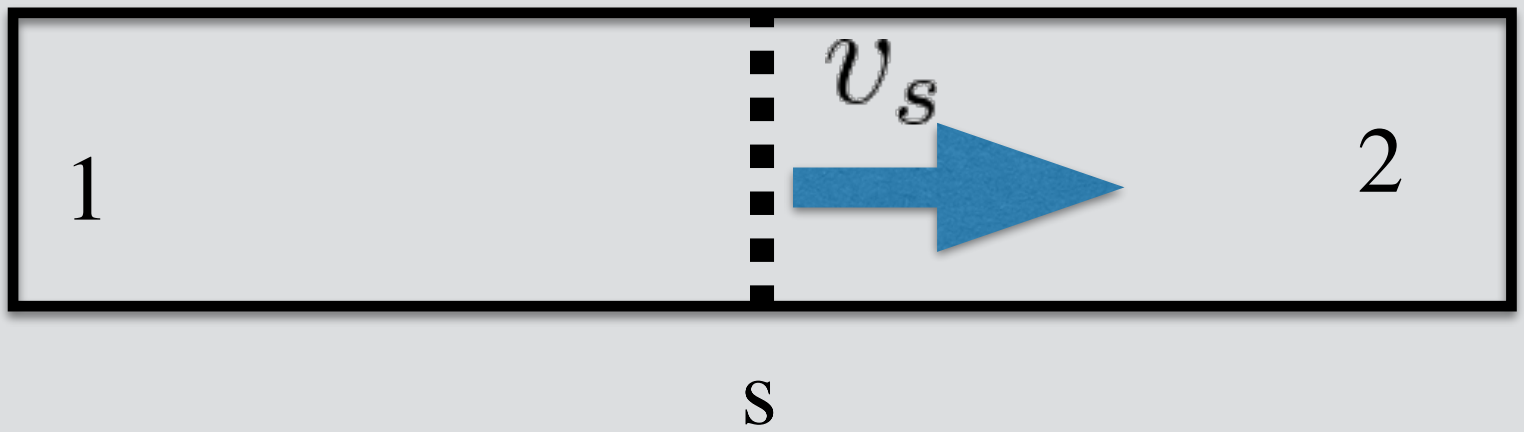
Governing Conservation Equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot F = S$$



Governing Conservation Equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot F = S$$



Change of mass in region 1 $= A_s(\rho_1 - \rho_2)v_s$

In steady state…

$$A_s(\rho_1 - \rho_2)v_s = F_1A_1 - F_2A_2 + \int S\,dV$$

In steady state…

$$A_s(\rho_1 - \rho_2)v_s = F_1 A_1 - F_2 A_2 + \int S \, dV$$

$$v_s \geq 0$$

In steady state…

$$A_s(\rho_1 - \rho_2)v_s = F_1 A_1 - F_2 A_2 + \int S\, dV$$

$$v_s \geq 0$$

$$F_1 A_1 - F_2 A_2 + \int S\, dV \geq 0$$

Energy equation $v_s \geq 0$

Energy equation $v_s \geq 0$

$$\left(\frac{h}{\phi}\right)_{\mathrm{NS}} \leq 1 + \frac{L_\nu \tau - C}{\dot{M}\phi_{\mathrm{NS}}}$$

Energy equation $v_s \geq 0$

$$\left(\frac{h}{\phi}\right)_{\mathrm{NS}} \leq 1 + \frac{L_\nu \tau - C}{\dot{M}\phi_{\mathrm{NS}}}$$

$$\Sigma \geq 0$$

Momentum equation $v_s \geq 0$

Momentum equation $v_s \geq 0$

$$x = \frac{r}{r_{\mathrm{NS}}} \qquad y = \frac{P}{\rho\phi} \qquad z = \frac{\rho}{\rho_{\mathrm{NS}}}$$

Momentum equation $v_s \geq 0$

$$y_1 + \int_1^{x_s} (2y - 1)\, z dx \geq 2 z_2 x_s$$

$$x = \frac{r}{r_{\rm NS}} \qquad y = \frac{P}{\rho\phi} \qquad z = \frac{\rho}{\rho_{\rm NS}}$$

Momentum equation $v_s \geq 0$

∇P

ρg

ρv^2

ram pressure

$$y_1 + \int_1^{x_s} (2y - 1)\, z\, dx \geq 2 z_2 x_s$$

$$x = \frac{r}{r_{\rm NS}} \qquad y = \frac{P}{\rho \phi} \qquad z = \frac{\rho}{\rho_{\rm NS}}$$

Momentum equation

$$v_s \geq 0$$

∇P

ρg

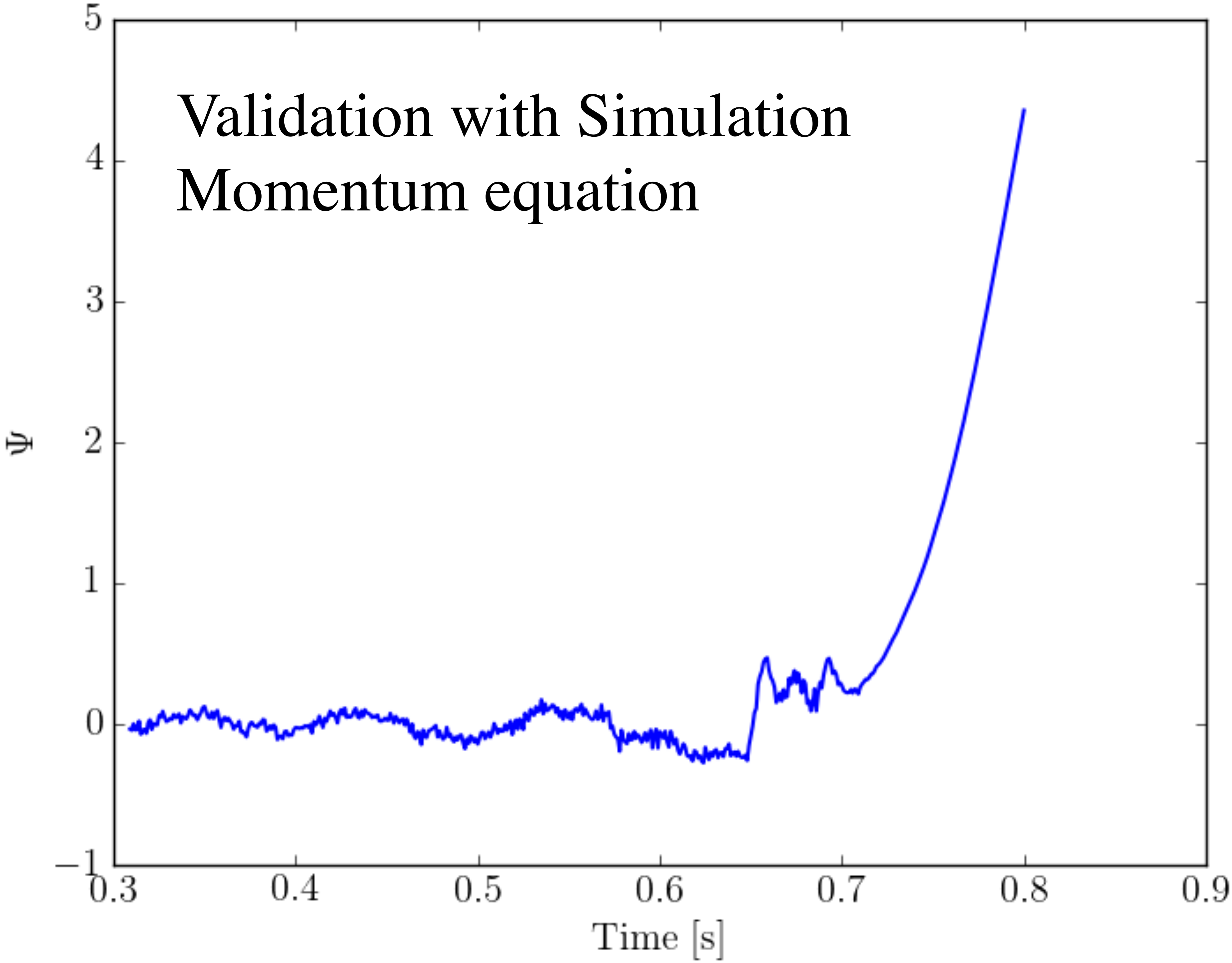
ρv^2

ram pressure

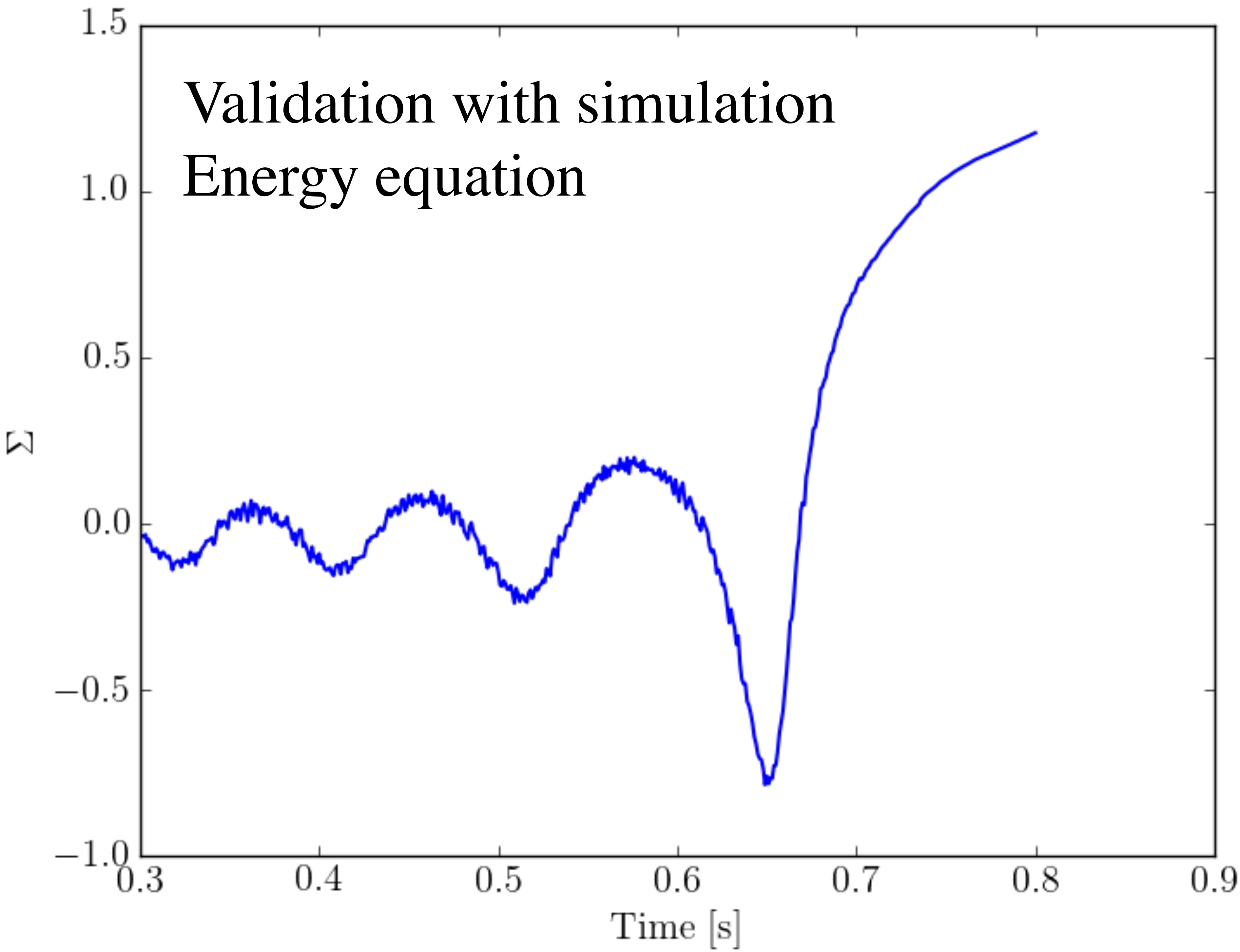
$$y_1 + \int_1^{x_s} (2y - 1)\, z\, dx \geq 2 z_2 x_s$$

$$\Psi \geq 0$$

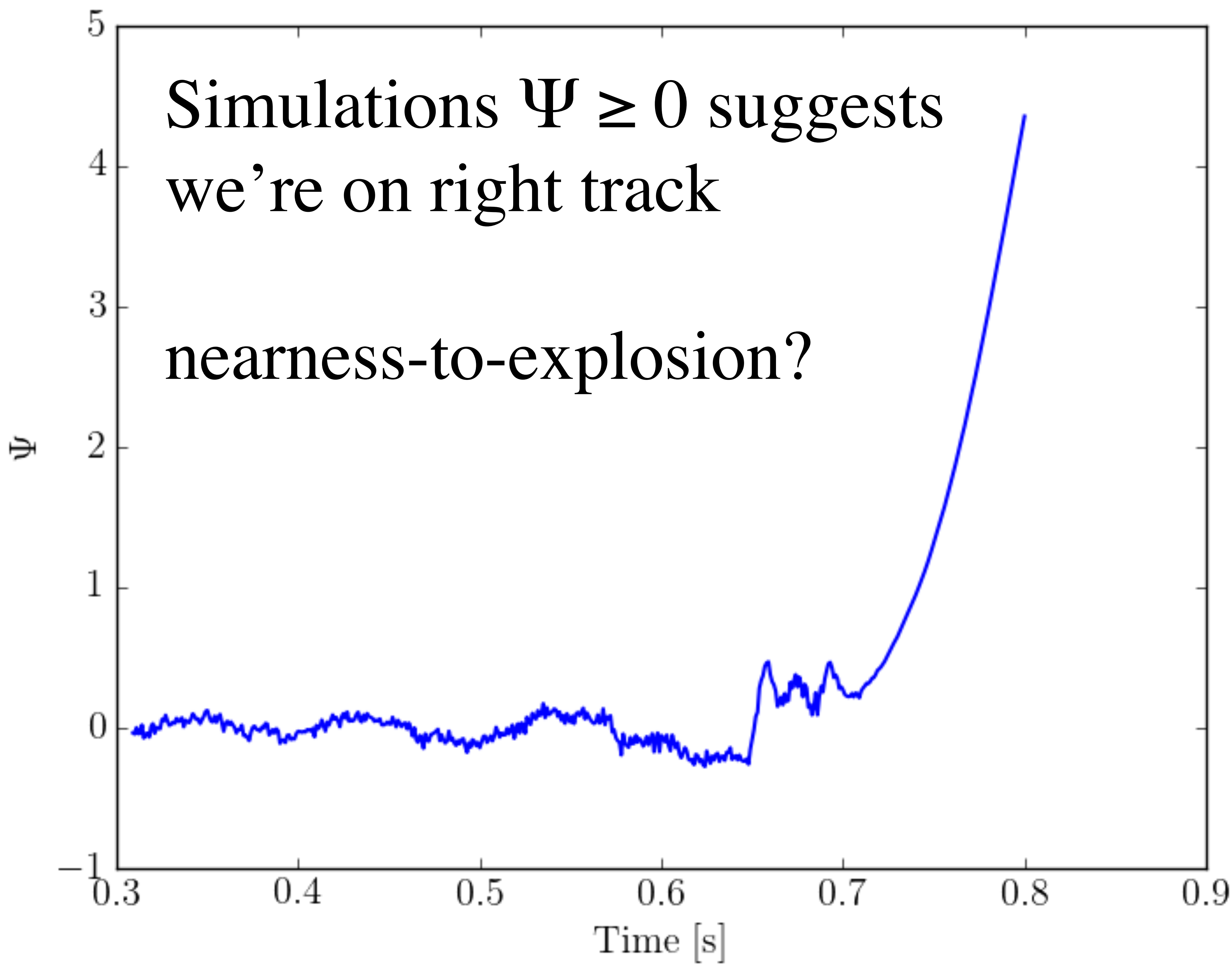
$$x = \frac{r}{r_{\mathrm{NS}}} \qquad y = \frac{P}{\rho \phi} \qquad z = \frac{\rho}{\rho_{\mathrm{NS}}}$$



Validation with Simulation
Momentum equation
Ψ
Time [s]
5
4
3
2
1
0
−1
0.3
0.4
0.5
0.6
0.7
0.8
0.9



Validation with simulation
Energy equation
1.5
1.0
0.5
0.0
−0.5
−1.0
Σ
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Time [s]



Simulations Ψ ≥ 0 suggests
we're on right track
nearness-to-explosion?
Ψ
Time [s]
5
4
3
2
1
0
−1
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Momentum equation $v_s \geq 0$

$$y_1 + \int_1^{x_s} (2y - 1)\, z dx \geq 2 z_2 x_s$$

Need Analytic expressions for

$$\Psi \geq 0$$

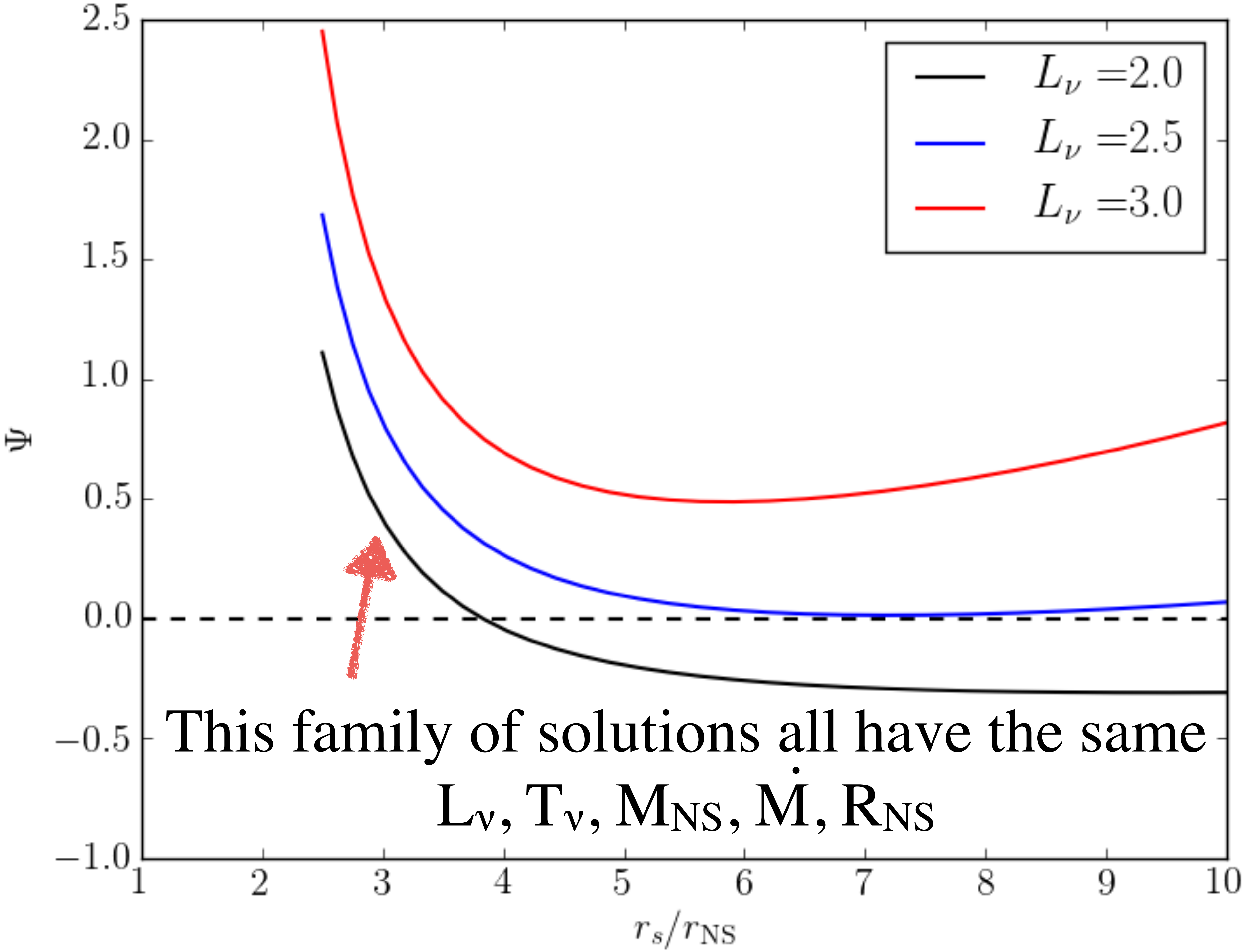
$$x = \frac{r}{r_{\rm NS}} \qquad y = \frac{P}{\rho\phi} \qquad z = \frac{\rho}{\rho_{\rm NS}}$$

Solution strategy:

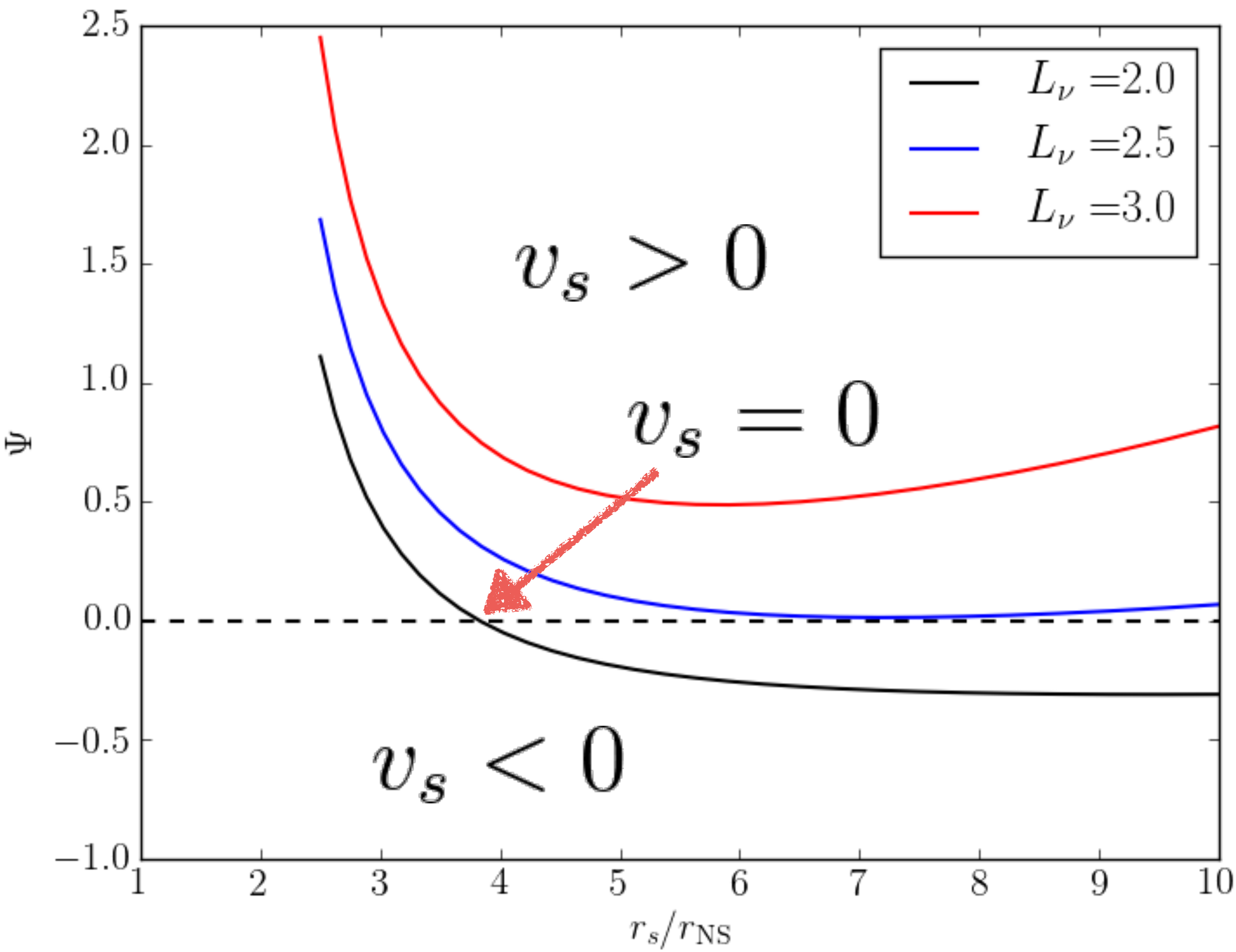
- Pick a trial R_s
- Find (semi-)analytic solution for y and z between R_{NS} and R_s
- Evaluate Ψ for this particular solution, it might be > 0, $= 0$, or < 0
- Repeat

Need Analytic expressions for

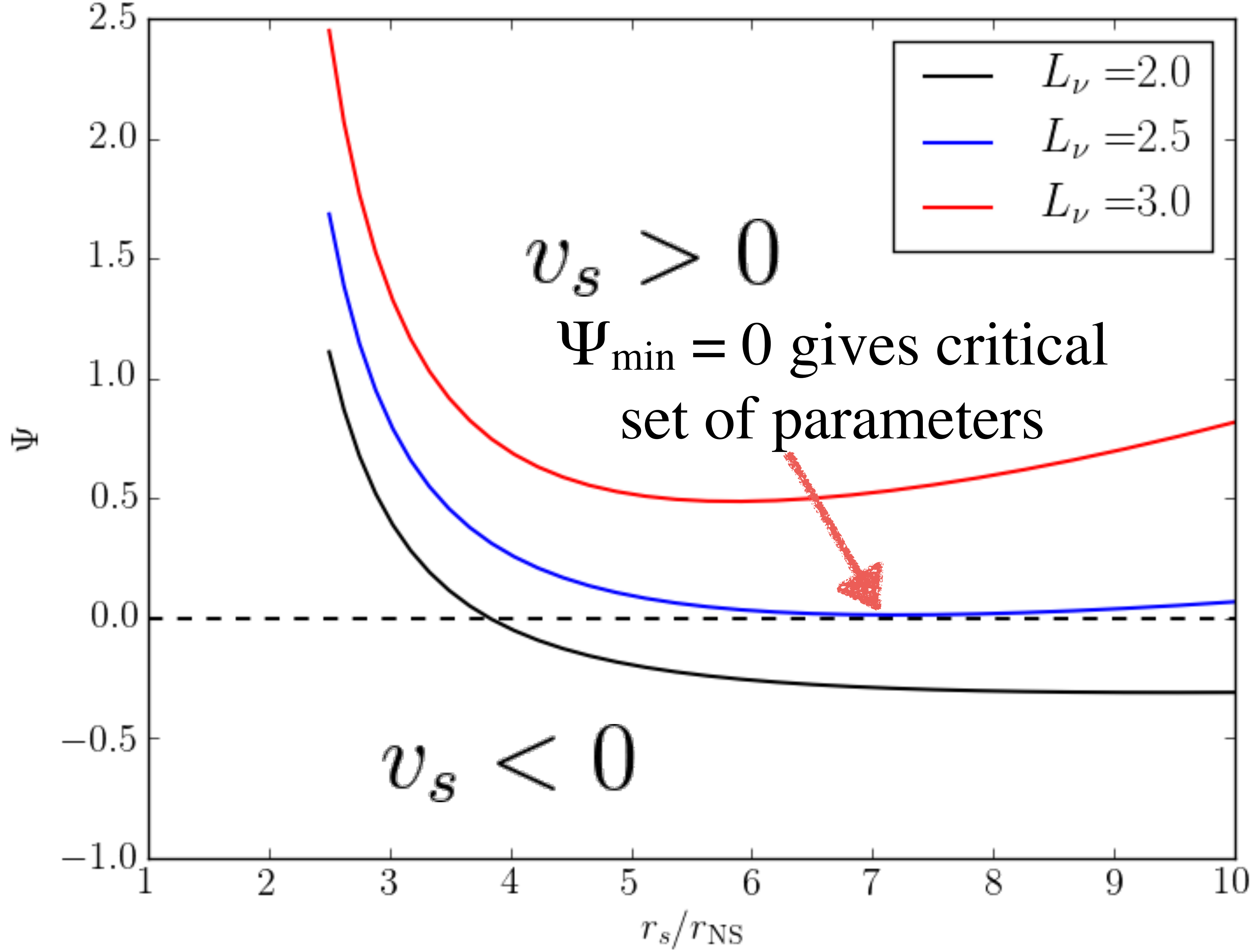
$$y = \frac{P}{\rho\phi} \qquad z = \frac{\rho}{\rho_{\rm NS}}$$



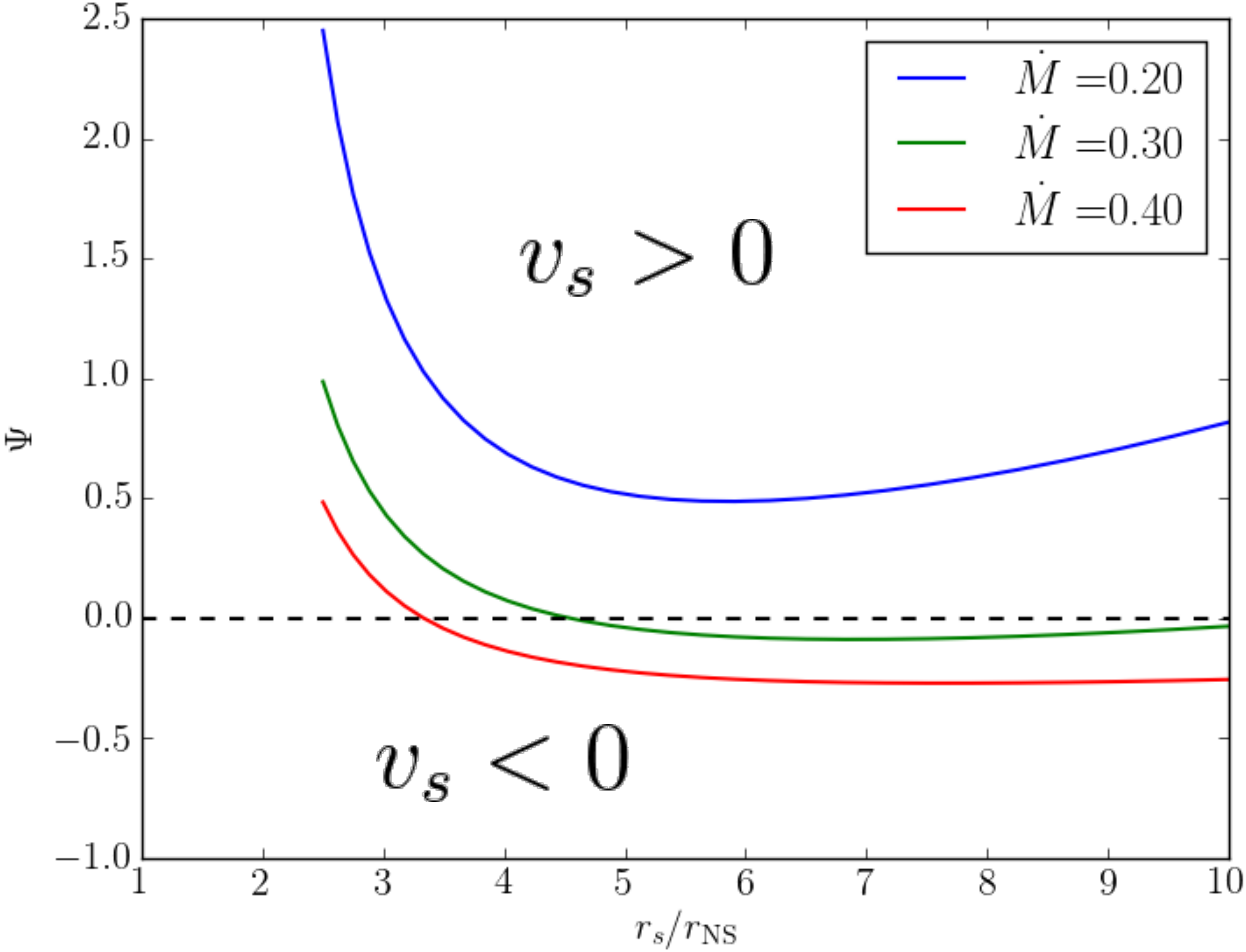
L_ν =2.0
L_ν =2.5
L_ν =3.0
Ψ
r_s/r_{NS}
This family of solutions all have the same
L_ν, T_ν, M_{NS}, $\dot{M}$, R_{NS}



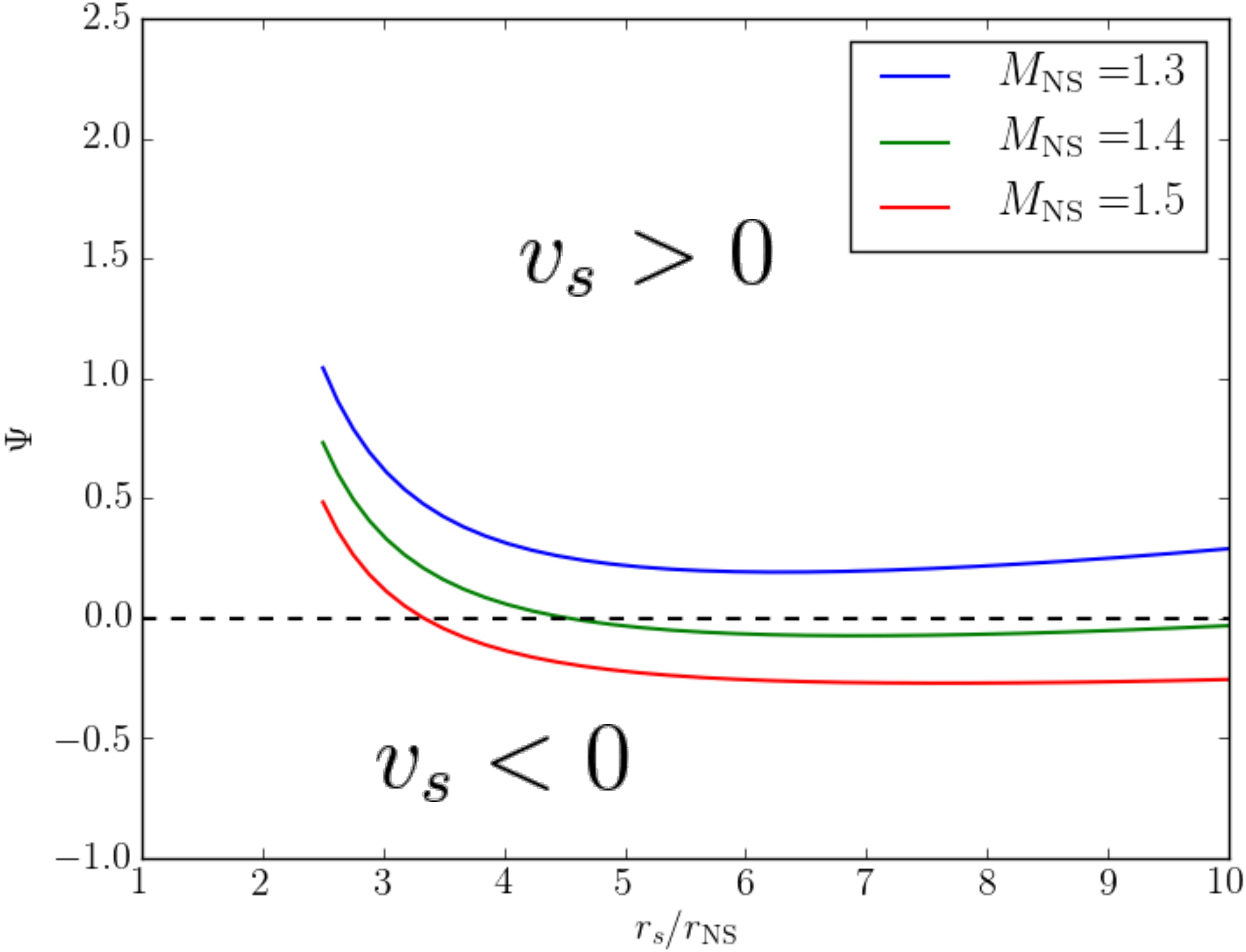
$L_\nu = 2.0$
$L_\nu = 2.5$
$L_\nu = 3.0$
$v_s > 0$
$v_s = 0$
$v_s < 0$
Ψ
$r_s / r_{\rm NS}$
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1
2
3
4
5
6
7
8
9
10



L_ν =2.0
L_ν =2.5
L_ν =3.0
$v_s > 0$
$\Psi_{min} = 0$ gives critical set of parameters
$v_s < 0$
Ψ
r_s/r_{NS}
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1
2
3
4
5
6
7
8
9
10

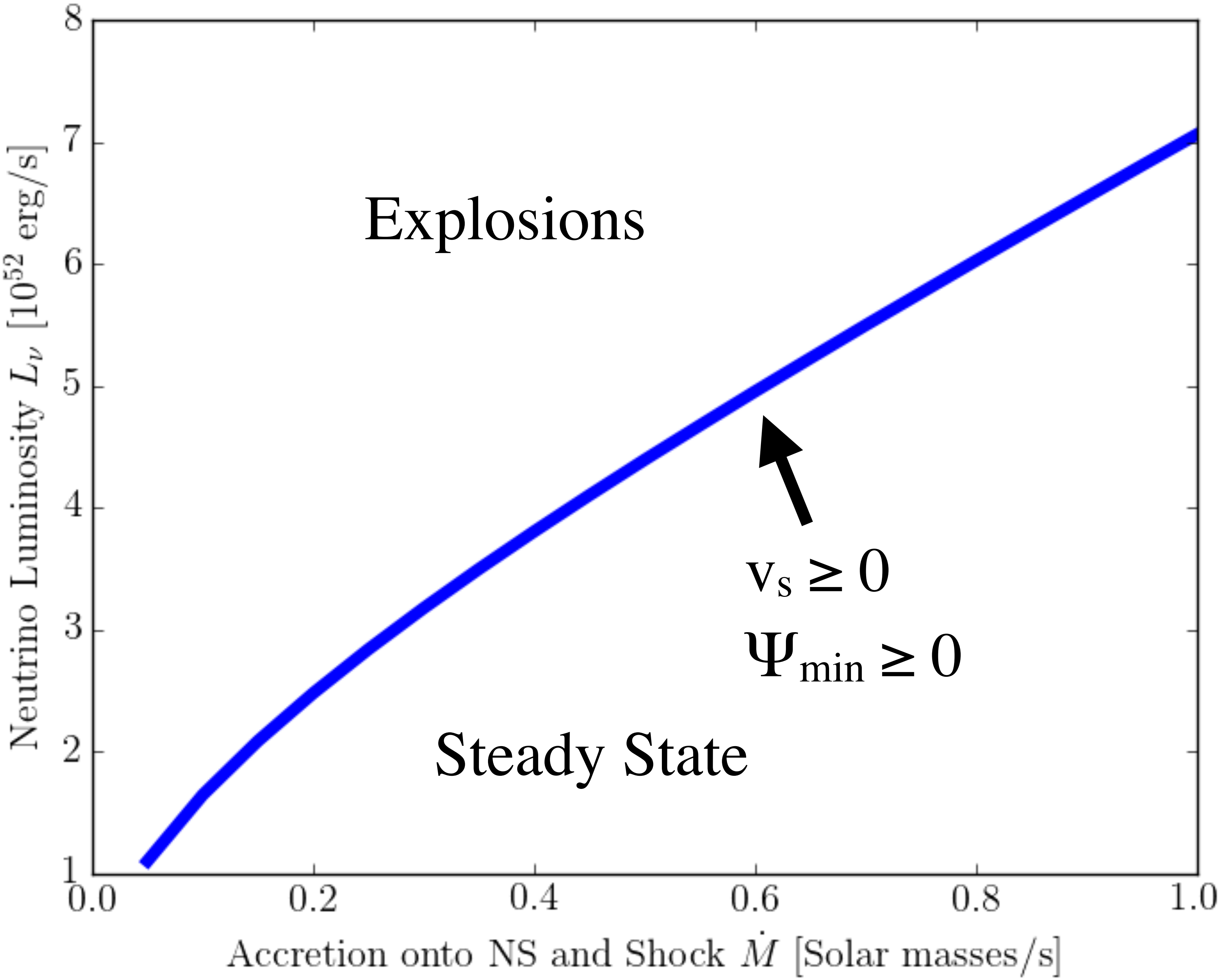


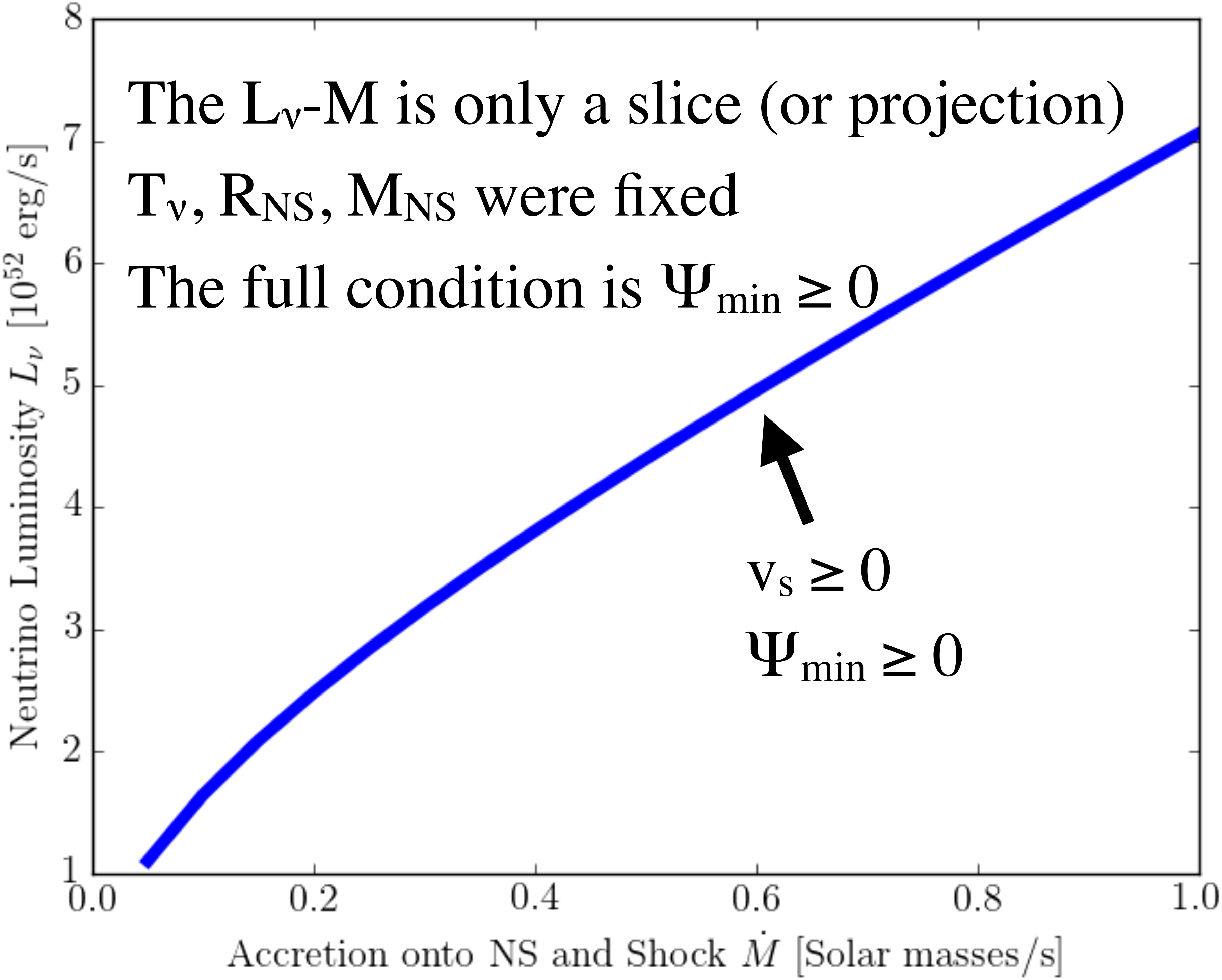
$\dot{M}$ =0.20
$\dot{M}$ =0.30
$\dot{M}$ =0.40
$v_s > 0$
$v_s < 0$
Ψ
$r_s/r_{\rm NS}$
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1
2
3
4
5
6
7
8
9
10



$M_{\rm NS}$ =1.3
$M_{\rm NS}$ =1.4
$M_{\rm NS}$ =1.5
$v_s > 0$
$v_s < 0$
Ψ
$r_s/r_{\rm NS}$
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1
2
3
4
5
6
7
8
9
10

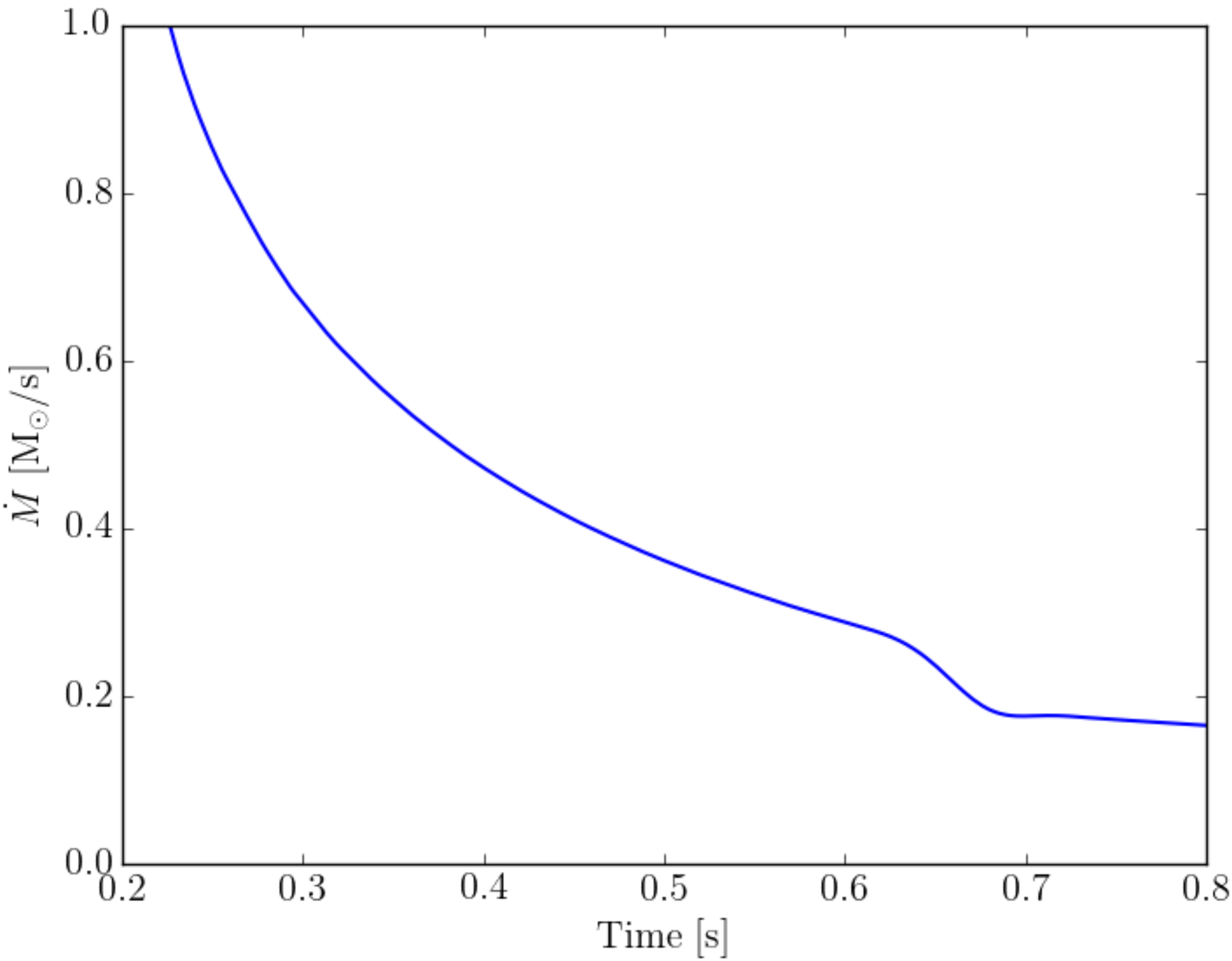
Use Ψ_{min} to derive L_ν-$\dot{M}$ critical curve

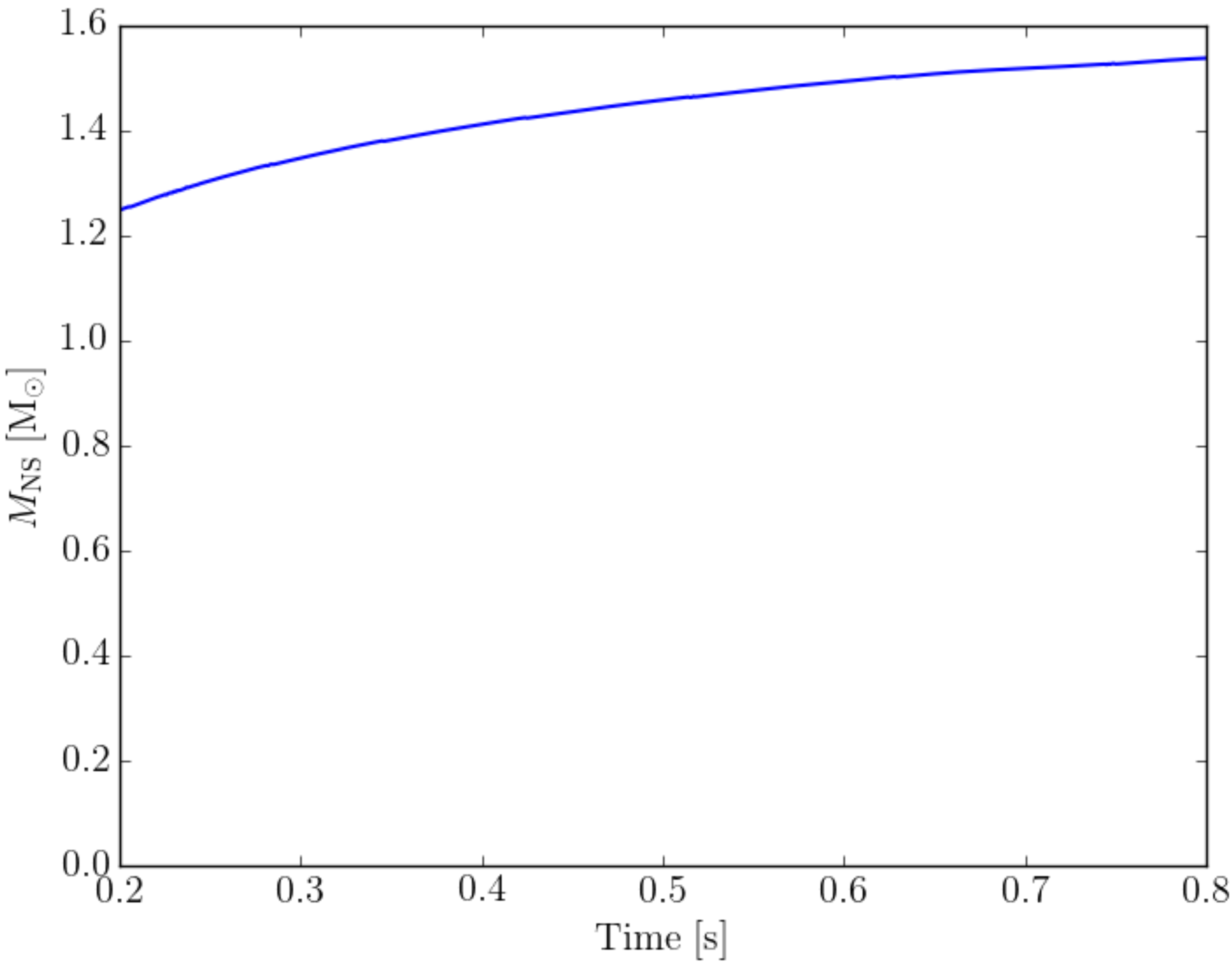

Explosions
Steady State
$v_s \geq 0$
$\Psi_{min} \geq 0$
Neutrino Luminosity L_ν [10^{52} erg/s]
Accretion onto NS and Shock $\dot{M}$ [Solar masses/s]
8
7
6
5
4
3
2
1
0.0
0.2
0.4
0.6
0.8
1.0



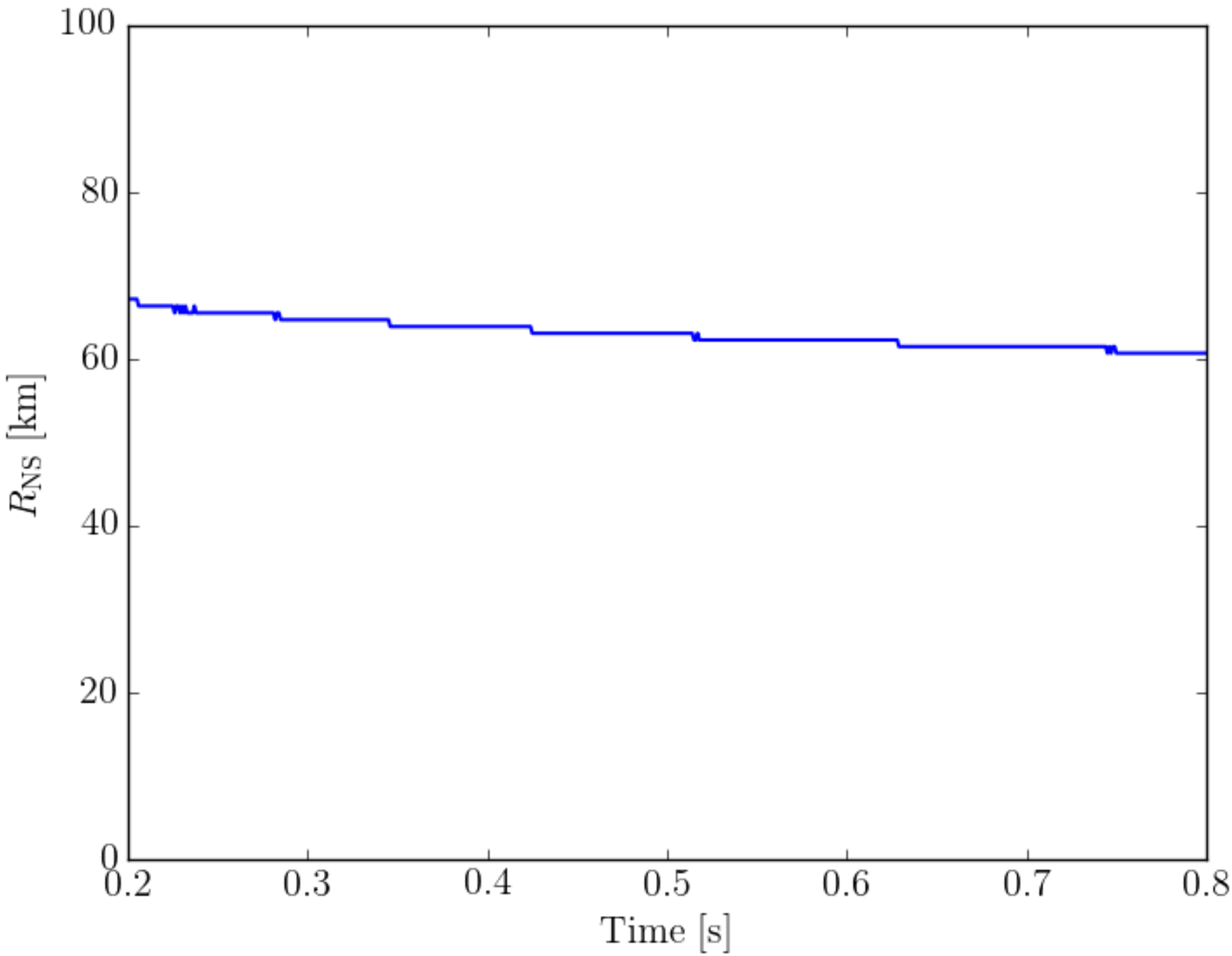
The L_ν-M is only a slice (or projection)
T_ν, R_{NS}, M_{NS} were fixed
The full condition is $\Psi_{min} \geq 0$
$v_s \geq 0$
$\Psi_{min} \geq 0$
Neutrino Luminosity L_ν [10^{52} erg/s]
Accretion onto NS and Shock $\dot{M}$ [Solar masses/s]
8
7
6
5
4
3
2
1
0.0
0.2
0.4
0.6
0.8
1.0

Use Ψ_{min} to evaluate nearness-to-explosion in 1D simulations

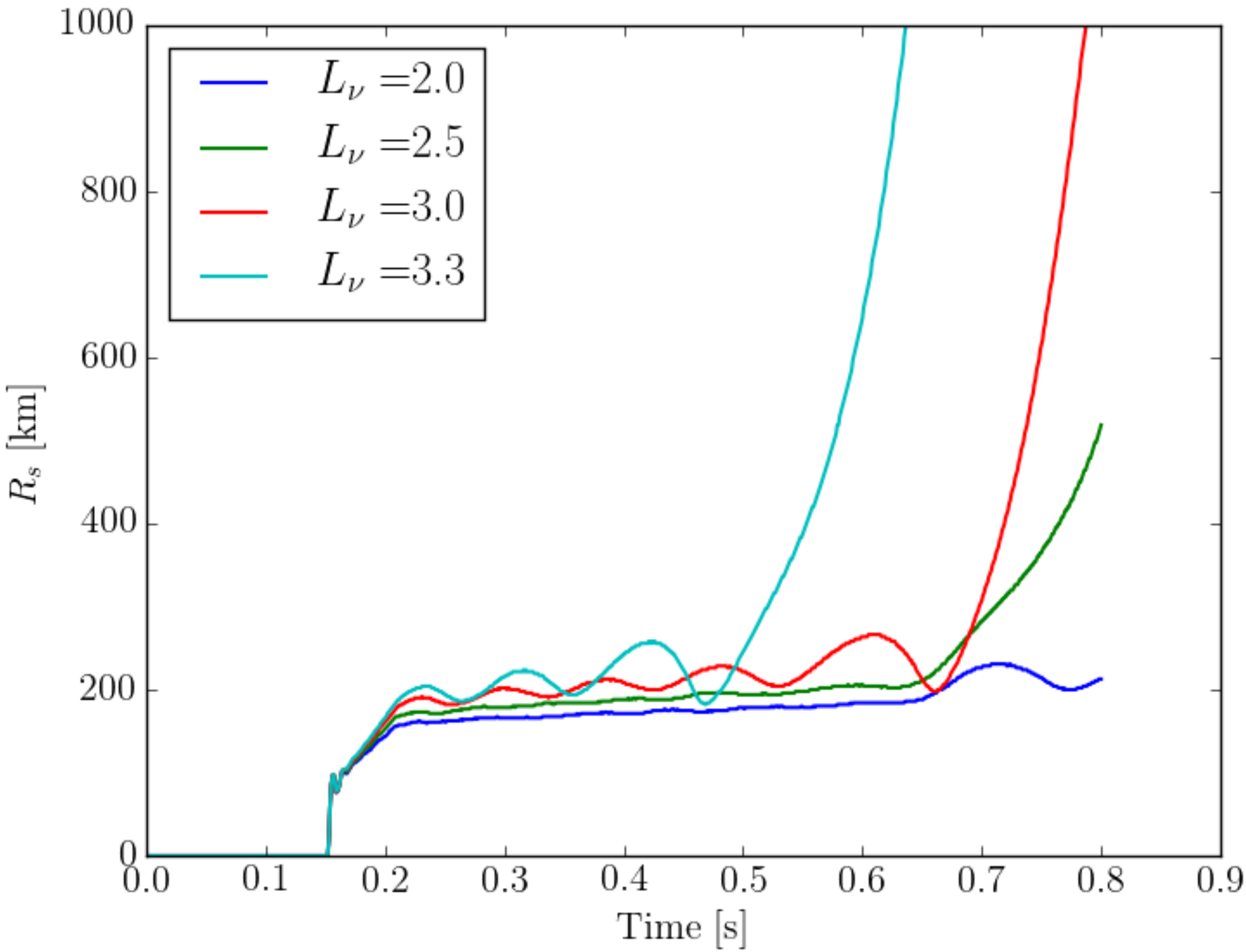

1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\dot{M}$ [$M_\odot$/s]
Time [s]



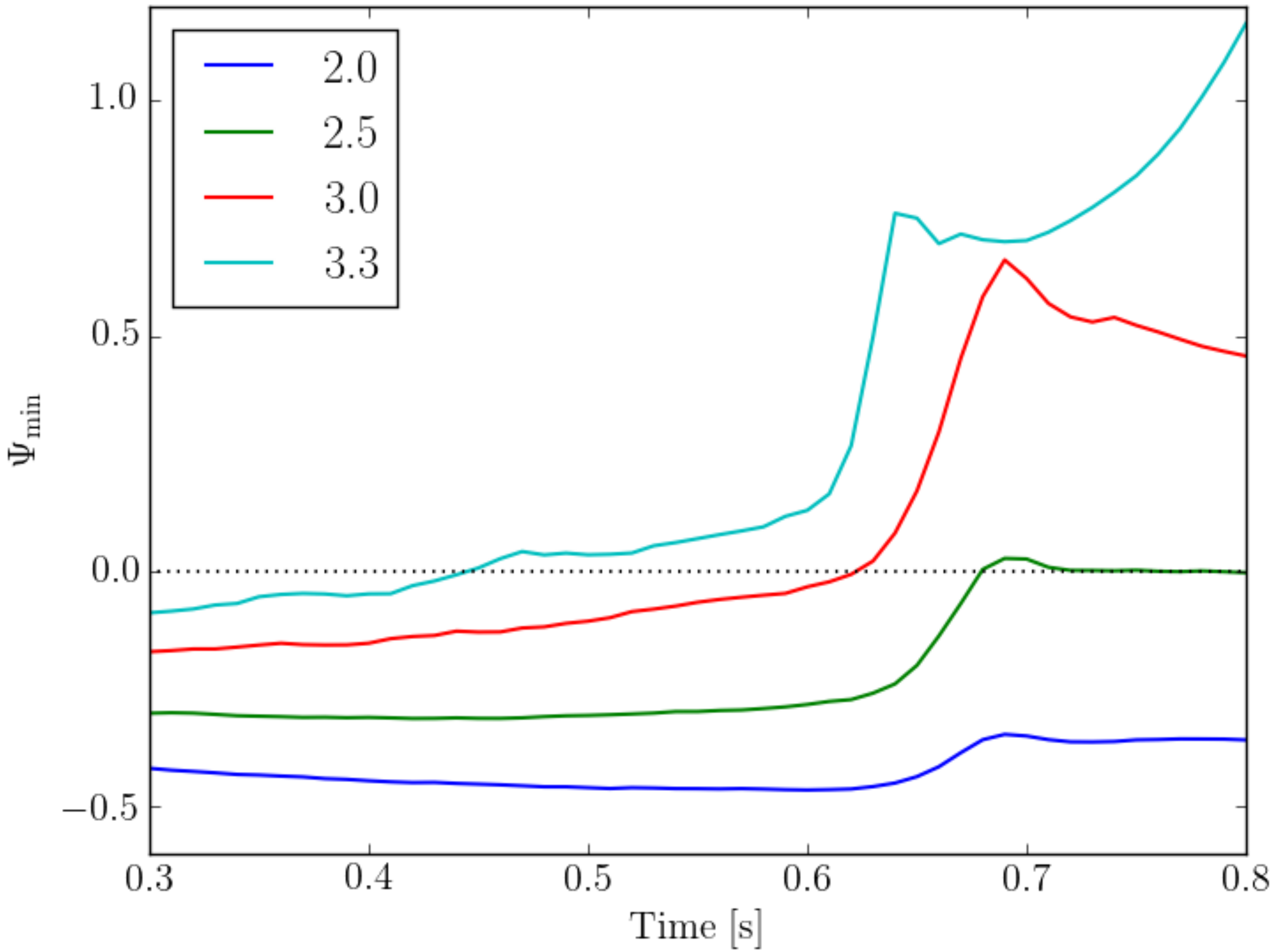
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
M_{NS} [$M_\odot$]
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Time [s]



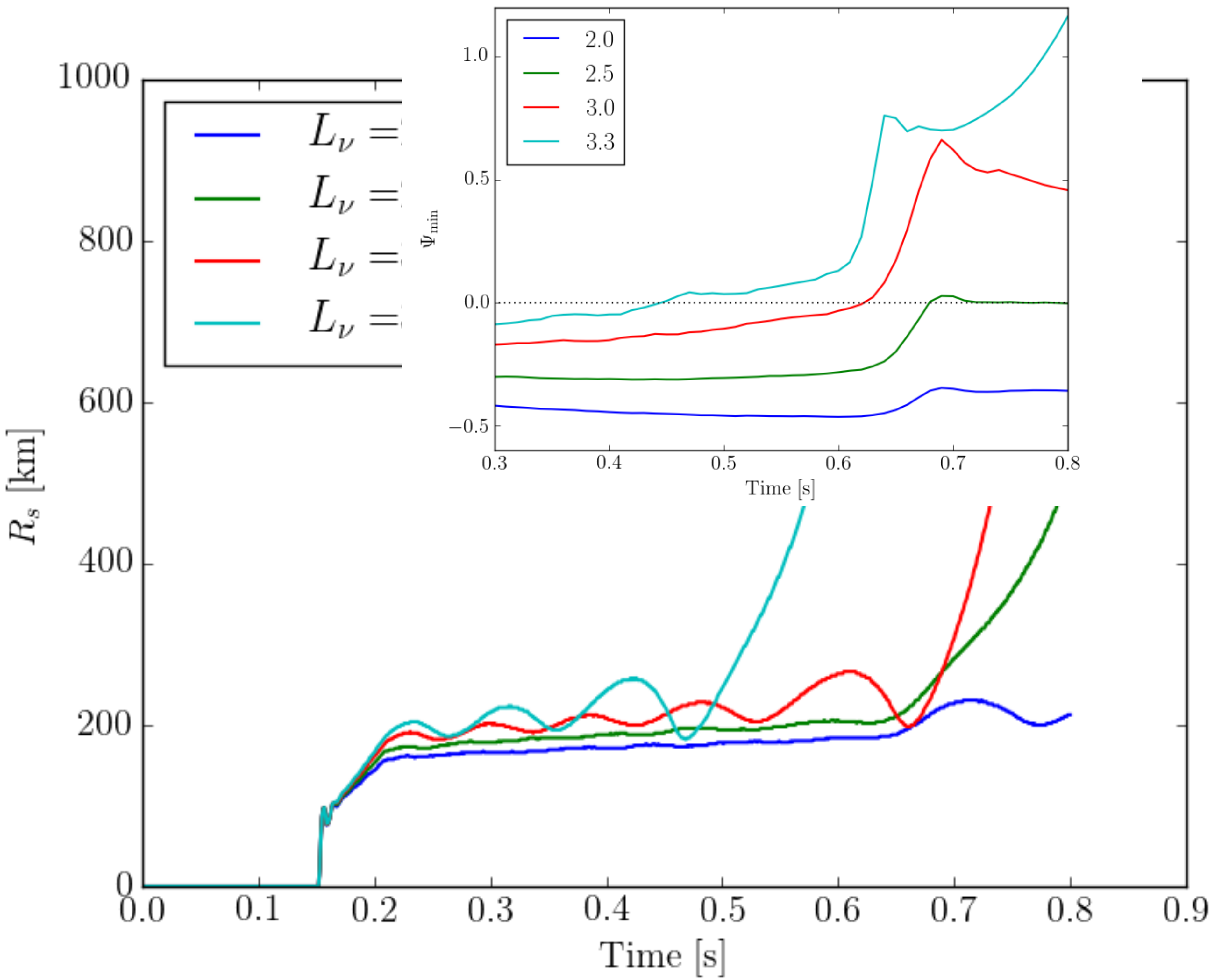
100
80
60
40
20
0
$R_{\rm NS}$ [km]
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Time [s]



L_ν =2.0
L_ν =2.5
L_ν =3.0
L_ν =3.3
R_s [km]
Time [s]
0
200
400
600
800
1000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9



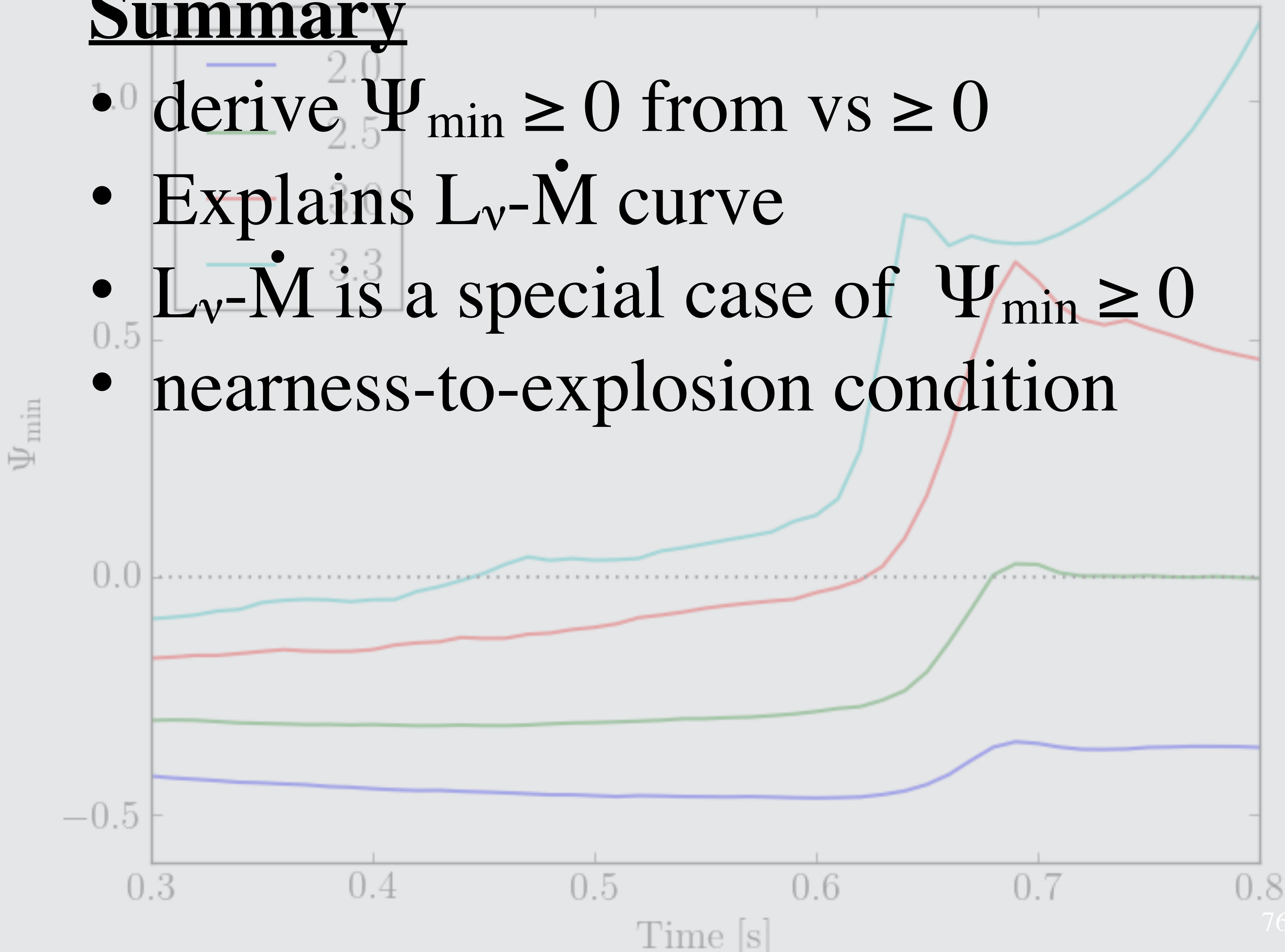
2.0
2.5
3.0
3.3
1.0
0.5
0.0
−0.5
$\Psi_{\min}$
0.3
0.4
0.5
0.6
0.7
0.8
Time [s]



1000
800
600
400
200
0
R_s [km]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Time [s]
$L_\nu =$
$L_\nu =$
$L_\nu =$
$L_\nu =$
2.0
2.5
3.0
3.3
1.0
0.5
0.0
−0.5
$\Psi_{\min}$
0.3
0.4
0.5
0.6
0.7
0.8
Time [s]

Summary

- derive $\Psi_{min} \geq 0$ from vs ≥ 0
- Explains L_ν-$\dot{M}$ curve
- L_ν-$\dot{M}$ is a special case of $\Psi_{min} \geq 0$
- nearness-to-explosion condition



Future Work

- Derive $\Psi_{min} \geq 0$ with turbulence
- Toward a truly analytic solution
- GR
- Compare with self-consistent 1D and 3D simulations
- Derive M_{NS}, $\dot{M}$, L_ν, and T_ν from progenitor