Comparing Sub-Grid Turbulence Models

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Modeling Turbulence

Resolving the full turbulent cascade is not possible in astrophysics simulations. **Sub-grid turbulence models** attempt to capture the effect of unresolved turbulence on the resolved flow.

Reynolds-Averaged Navier-Stokes (RANS) decompose the flow into mean and fluctuating components and model the majority of the cascade.

Large Eddy Simulations (LES) use a spatial filtering technique to decompose the flow into resolved and unresolved components. Only the smallest scales are modeled.

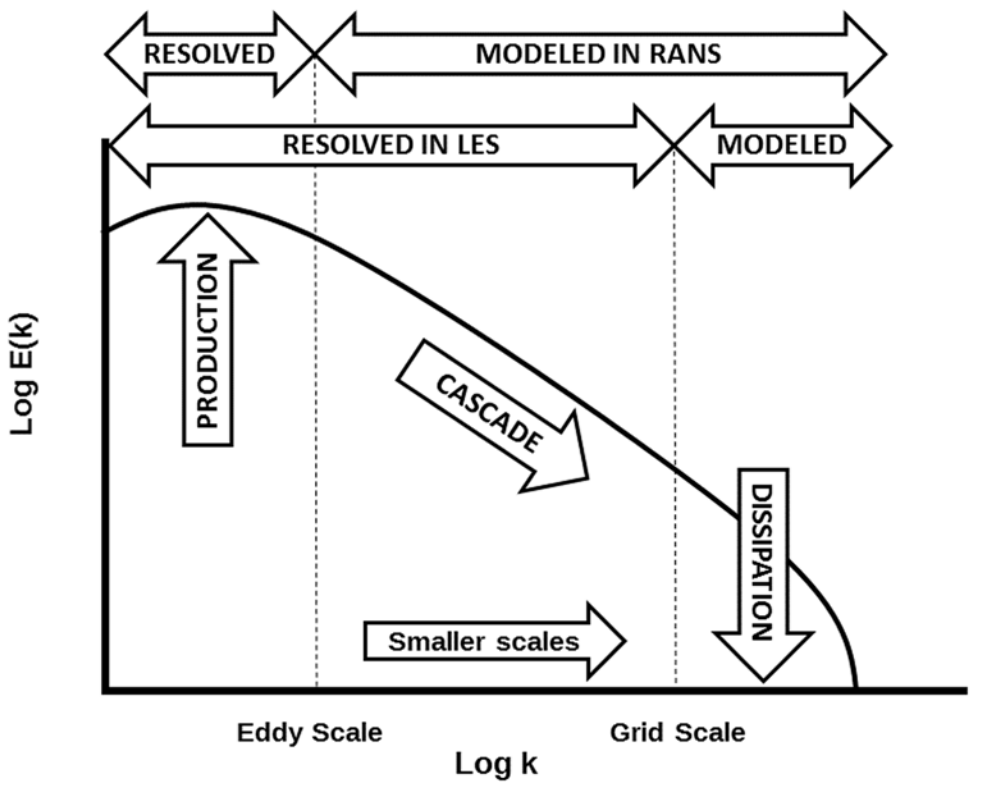

Figure 1. Schematic power spectrum representing the scales of different modeling methods.

Eddy Viscosity

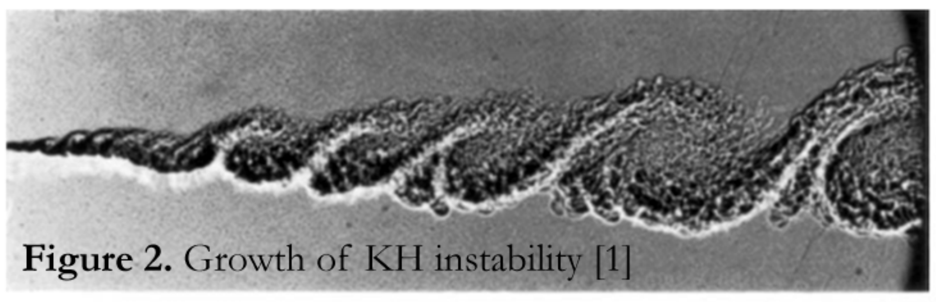

Figure 2. Growth of KH instability [1]

Turbulent eddies transport energy to smaller scales where it is dissipated as heat. In this way **turbulence resembles molecular dissipation**, and it is modeled with an analogous **eddy viscosity** and **turbulent stress tensor**.

$$\mu_T = C_\mu \rho \frac{k^2}{\epsilon} = C_\mu \rho L \sqrt{k} = C_\mu \rho \frac{k}{\omega}$$

$$\tau_{ij}^T = \mu_T\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu_T \delta_{ij} \frac{\partial u_k}{\partial x_k} - 2C_T \rho k \frac{u_{i,k} u_{j,k}}{u_{m,n} u_{m,n}} - \frac{2}{3}(1 - C_2)\delta_{ij} \rho k$$

Future Work

- Compressibility corrections for RANS
- Buoyancy terms for k-ϵ
- K-H calibration for k-L
- Dynamic filtering for LES
- AMR for LES

Turbulence Models

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_i}\left(\frac{\mu_T}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \tau_{ij}^T \frac{\partial u_i}{\partial x_j} - C_D \rho \epsilon - C_B \rho k A_i g_i$$

k : Turbulent energy

Diffusion **Shear (Kelvin-Helmholtz)** **Dissipation** **Buoyancy (Rayleigh-Taylor) (Only k-L)**

LES[2] Uses grid as filter. Highly compressible. Poor for organized and under-resolved flow.

k-ϵ [3] $\in$: *Dissipation rate*. Extensively tested. Good for subsonic shear flows.

k-ω [4] ω : *Dissipation frequency*. Good for shear flows. Sensitive to initial conditions.

k-L [5] L : *Eddy scale length*. Adds buoyancy-drag for R-T instability. Not calibrated for K-H.

Shock-Cloud Interaction

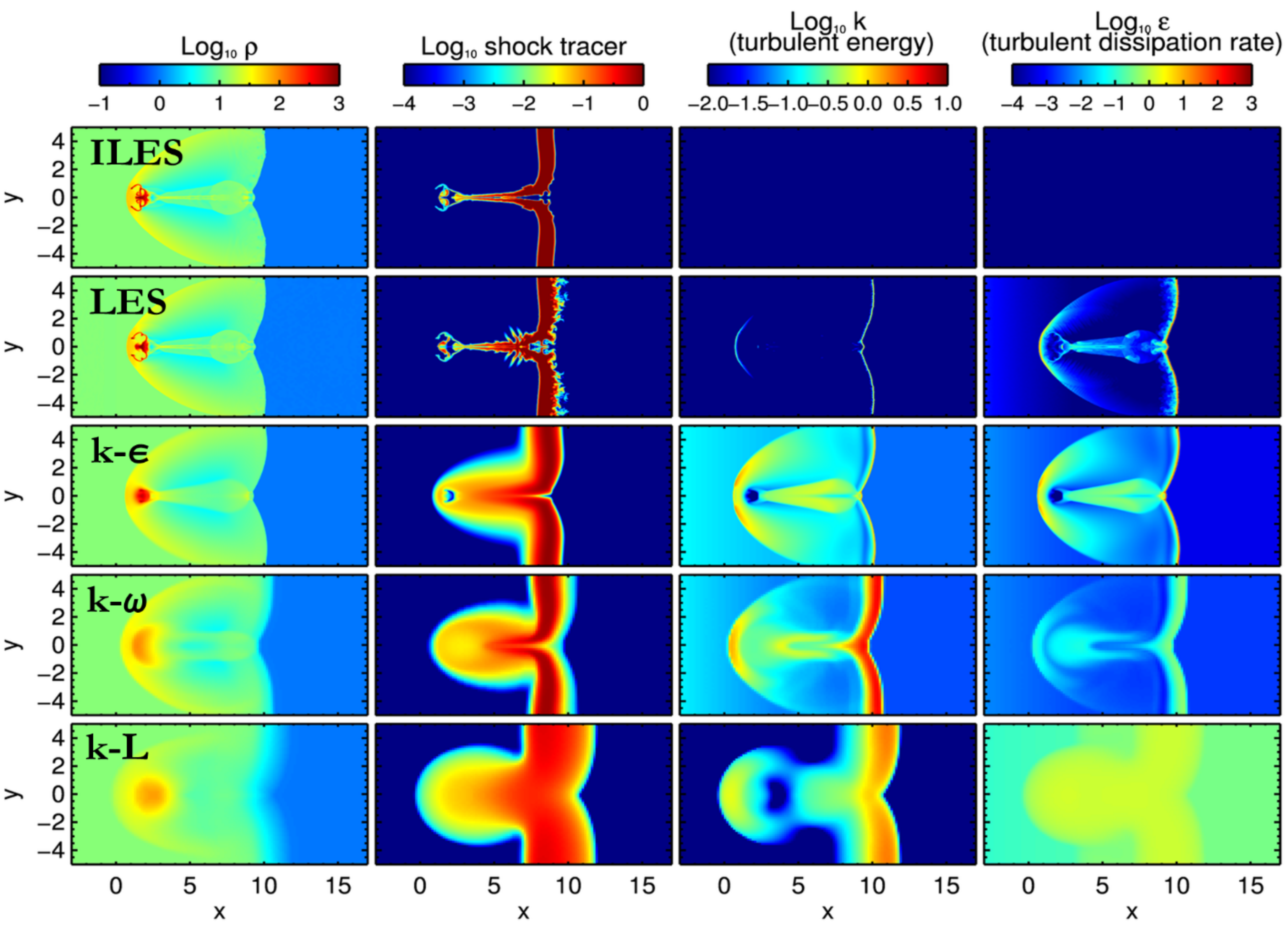

Figure 3. Snapshot from the shock-cloud interaction after 1 dynamical timescale. A supersonic shock wave impacts a dense cloud. We implement the turbulence models in the **Athena hydrodyanmics code [6]**.

Reduction of Bottleneck Effect

Numerical viscosity can serve as an implicit LES model but underestimates the dissipation at small scales. This pile-up of kinetic energy is known as the "**bottleneck effect**." The explicit viscosity of LES alleviates the bottleneck. We test the LES model with a decaying adiabatic turbulence simulation [7].

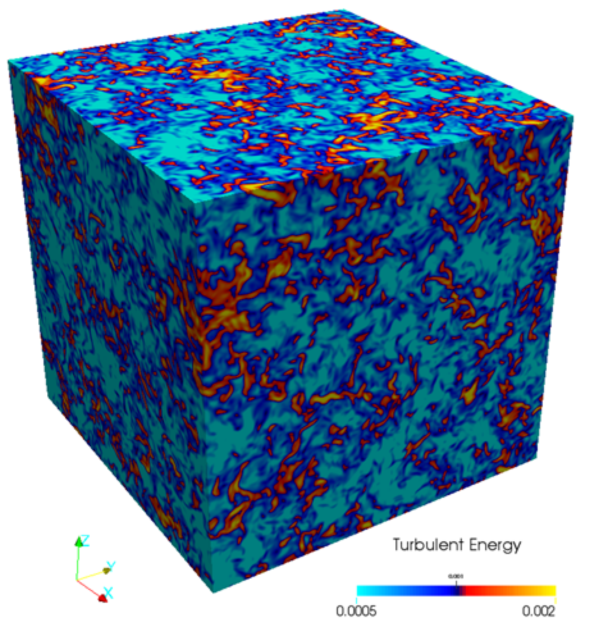

Figure 4. Visualization of turbulent kinetic energy after 10 decay times.

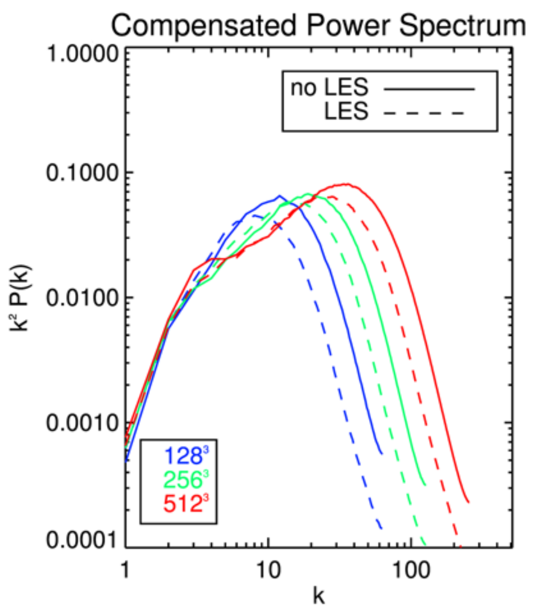

Figure 5. Compensated power spectrum. The LES model reduces the excess energy at small scales.

[1] Brown, G. and Roshko, A. 1974, J. Fl. Mech., 64, 775
[2] Schmidt, W. and Federrath, C. 2011, A&A, 528A, 106
[3] Launder, B. and Spalding, D. 1974, Comput. Method Appl. M., 3, 269
[4] Wilcox, D. C., *Turbulence Modeling for CFD*, 1993
[5] Dimonte, G. and Tipton, R. 2006, Phys. Fluids, 18, 085101
[6] Stone, J. M. et al. 2008, ApJS, 178, 137
[7] Woodward, P. et al. 2006, JPhCS, 46, 370

Supported by NSF Grant AST-1109085 and NC Space Grant