Angular Momentum Transport via Waves in Massive Stars

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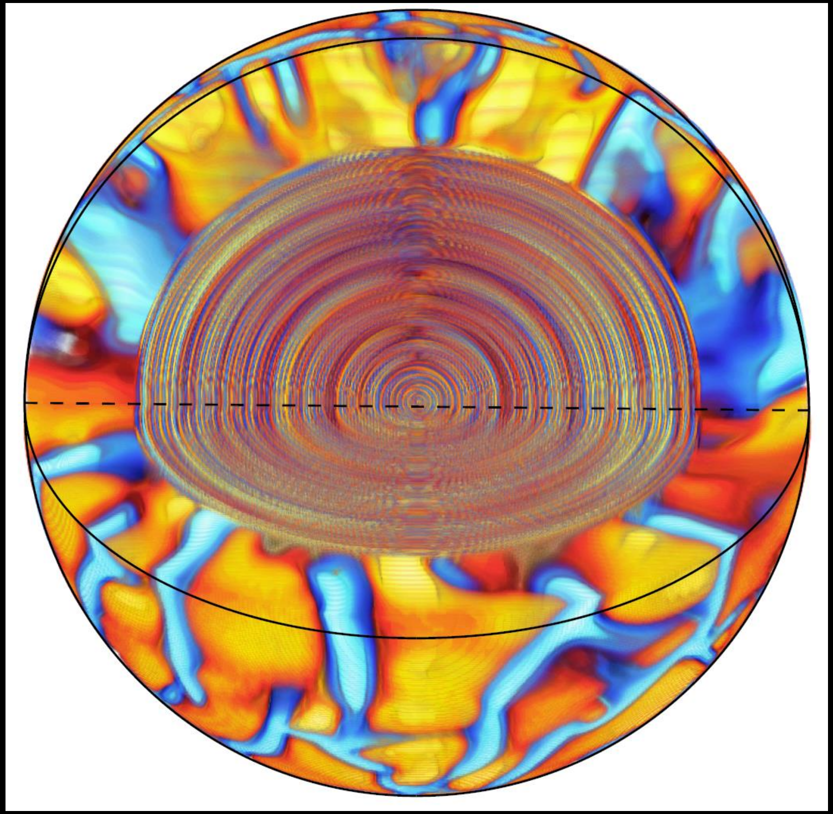


Alvan et al. 2014

Collaborators: Matteo Cantiello, Daniel Lecoanet, Eliot Quataert

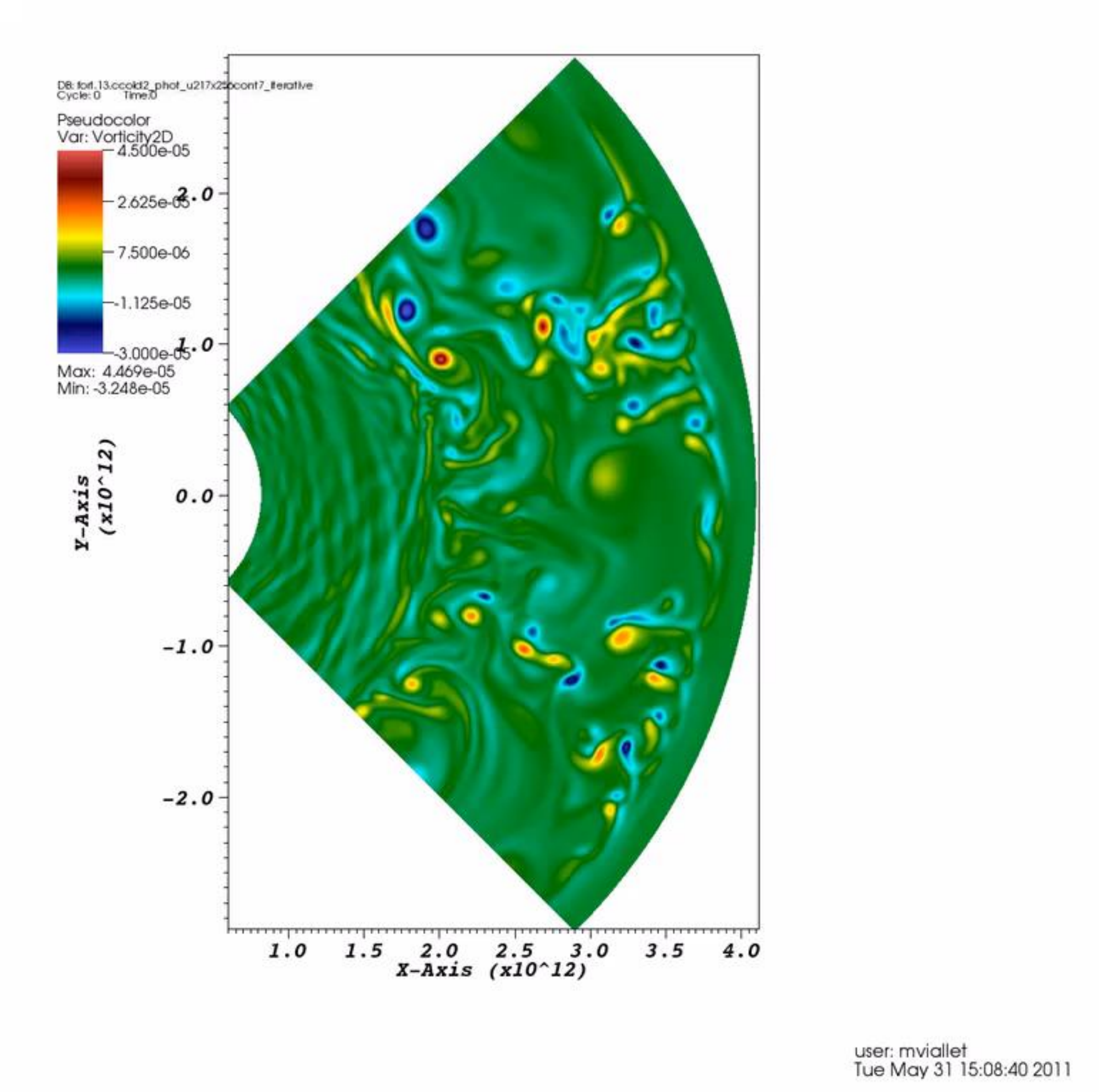
Motivation

- Rotation rate at core collapse is important

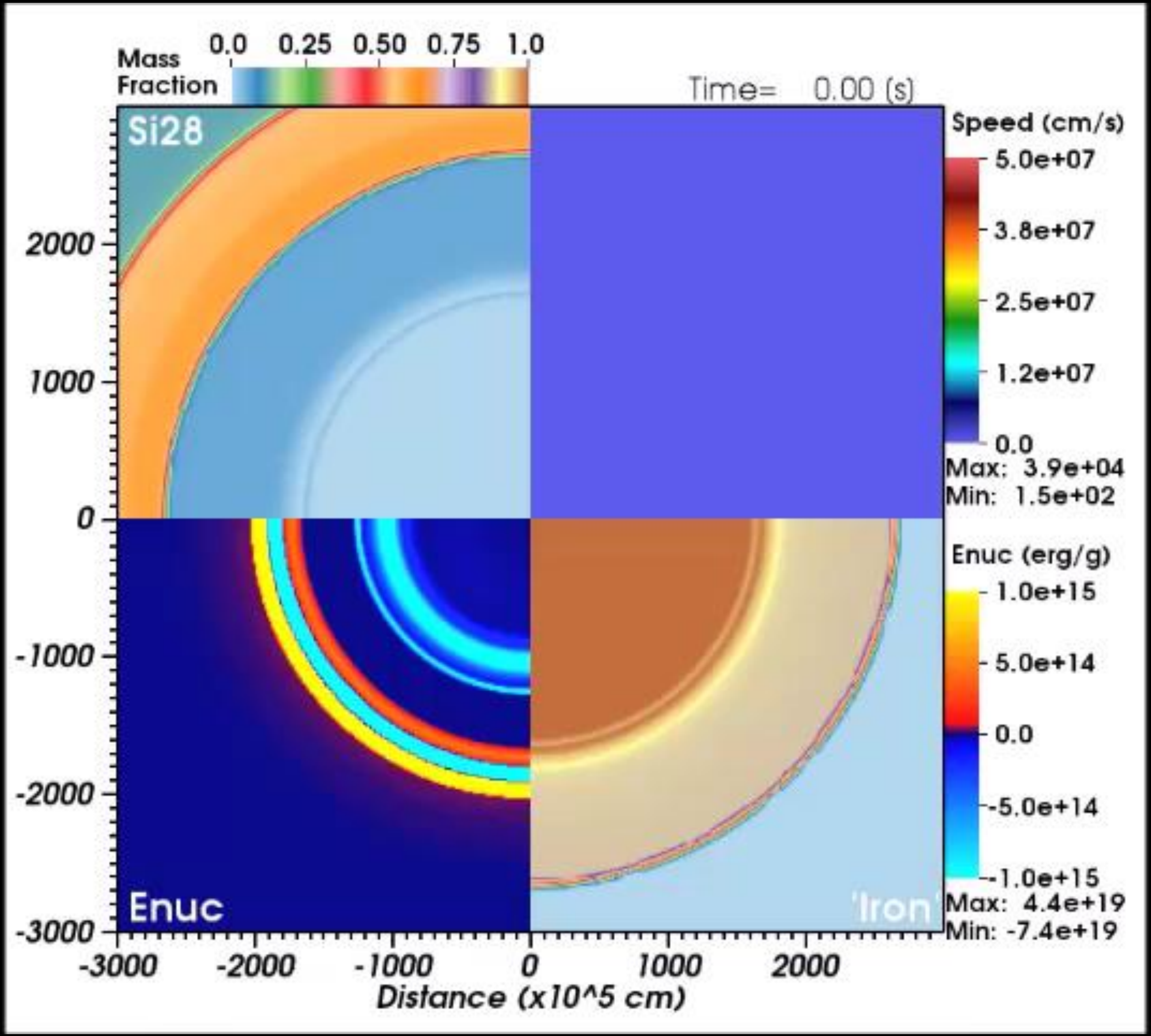
    - May determine nature of supernova
        - GRBs, SLSNe may require rapid rotation

    - May determine rotation rate of compact object
        - Neutron stars rotate slowly ($P_i > 10$ ms)
        - Black holes rotate rapidly

- Waves carry huge amount of energy during late burning stages



Courtesy of Dave Arnett



Courtesy of Sean Couch

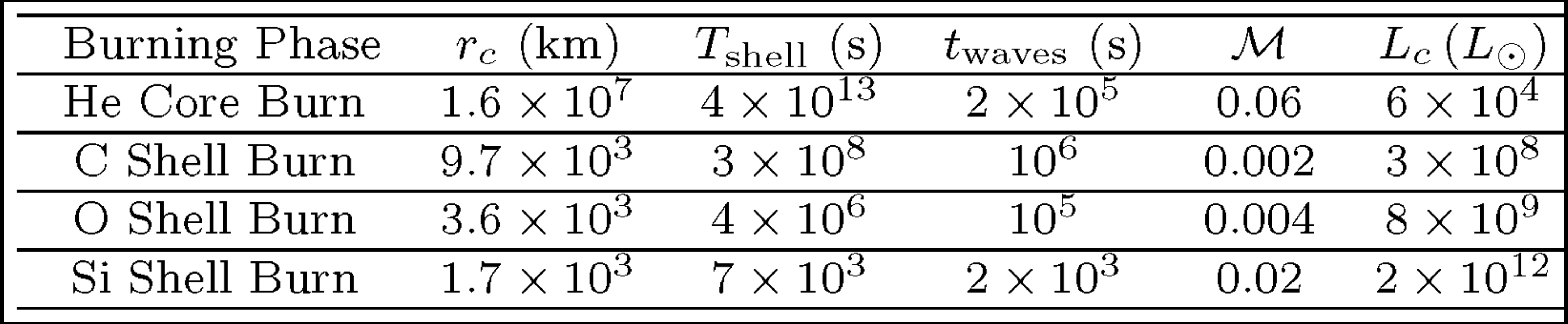
Internal gravity waves excited by convective motions, propagate through adjacent radiative zones

Power of Waves

- Surface binding energy is tiny
  - Wave power is very super-Eddington
  - Waves may induce pre-SN mass loss (Quataert & Shiode 2012)

- Core moment of inertia is tiny
  - Waves may substantially change spin rate

$$\dot{E} \sim \mathcal{M} L$$



Burning Phase	r_c (km)	$T_{\rm shell}$ (s)	$t_{\rm waves}$ (s)	$\mathcal{M}$	$L_c\,(L_\odot)$
He Core Burn	1.6×10^7	4×10^{13}	2×10^5	0.06	6×10^4
C Shell Burn	9.7×10^3	3×10^8	10^6	0.002	3×10^8
O Shell Burn	3.6×10^3	4×10^6	10^5	0.004	8×10^9
Si Shell Burn	1.7×10^3	7×10^3	2×10^3	0.02	2×10^{12}

Energy and Angular Momentum Budget

- Waves carry energy into radiative zone at rate

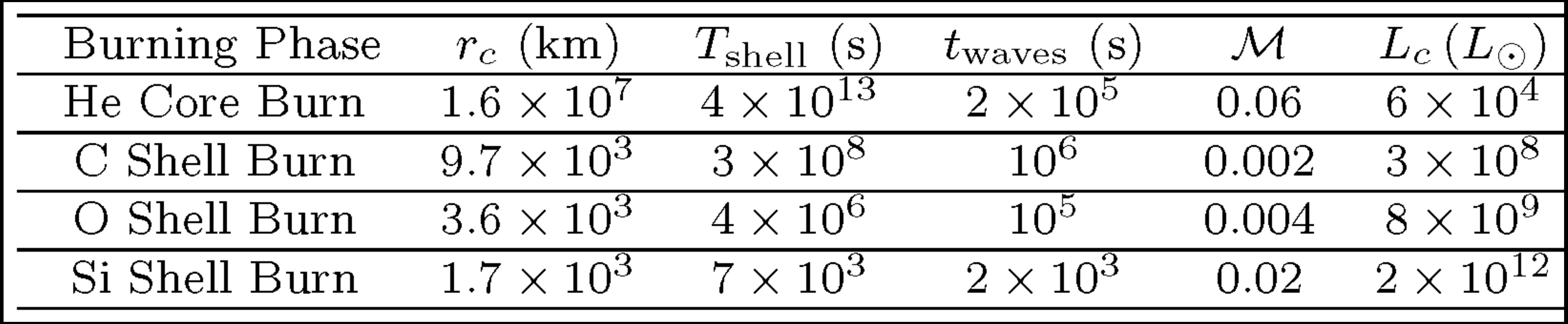
$$\dot{E} \sim \mathcal{M} L$$

- The corresponding angular momentum flux is

$$\dot{J} \sim \frac{m_c}{\omega_c} \dot{E}$$

- A characteristic time scale on which waves *could* change the spin of the radiative region is

$$t_*(r) = \frac{J(r)}{\dot{J}(r)}$$



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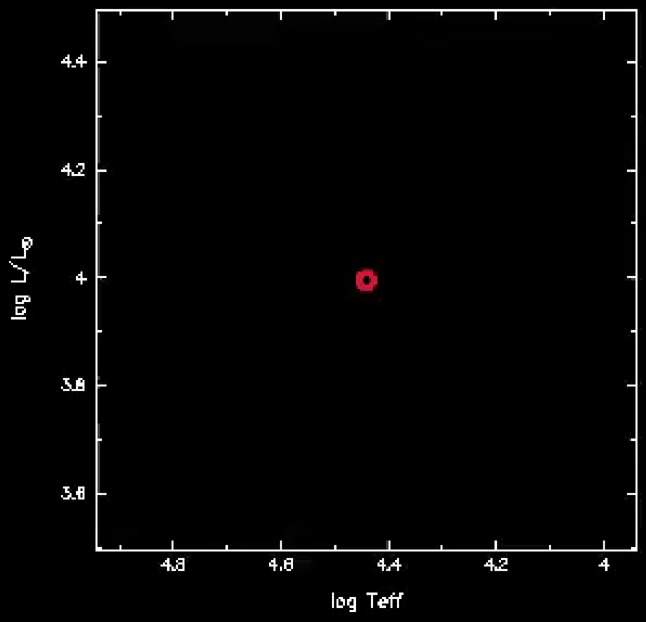
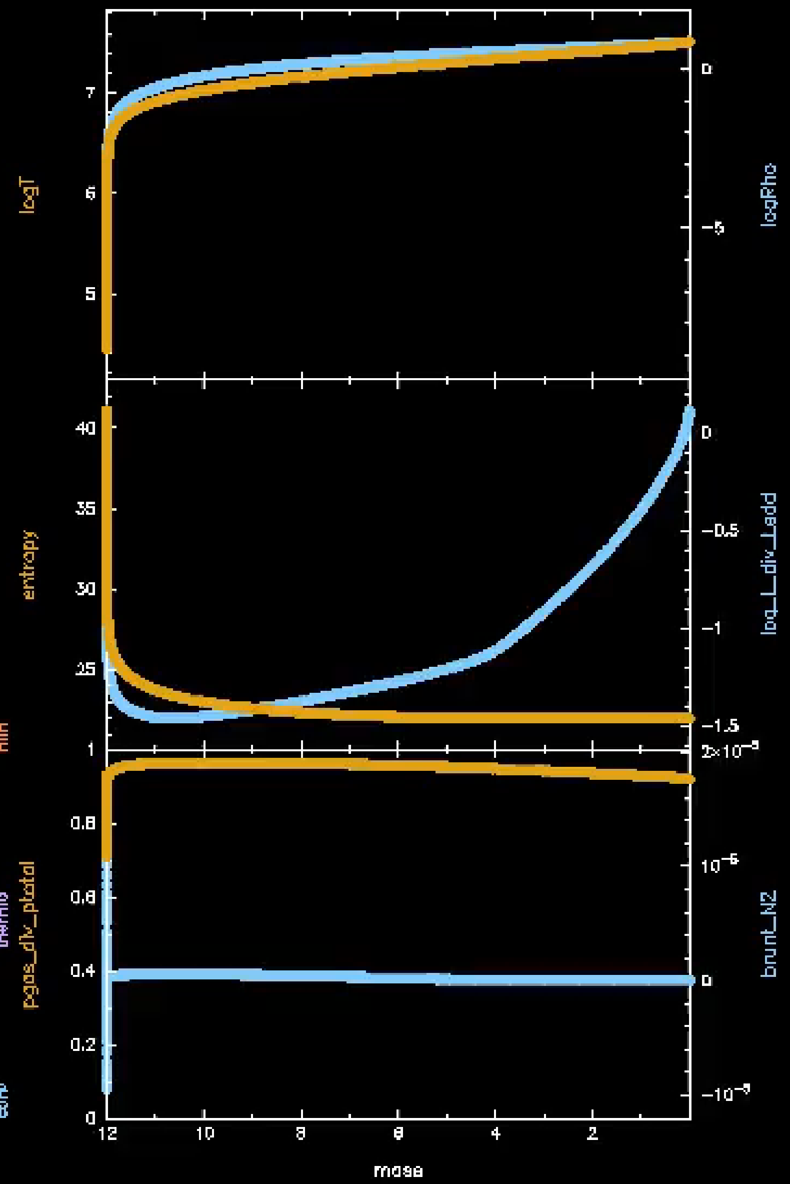
```
age 6.326426e9 secs                                                                                                     model 1
            time_step      200.4888435            num_zones    915              star_mass     11.9999999        star_mdot   -9.870E-10
log_total_angular_momentum -99.0000000         center_omega    0.0000000    surf_avg_omega    0.0000000    surf_avg_v_rot    0.0000000
             log_Teff      4.4413414          photosphere_L    9887.3100097  photosphere_r    4.3462533       c_core_mass    0.0000000
```

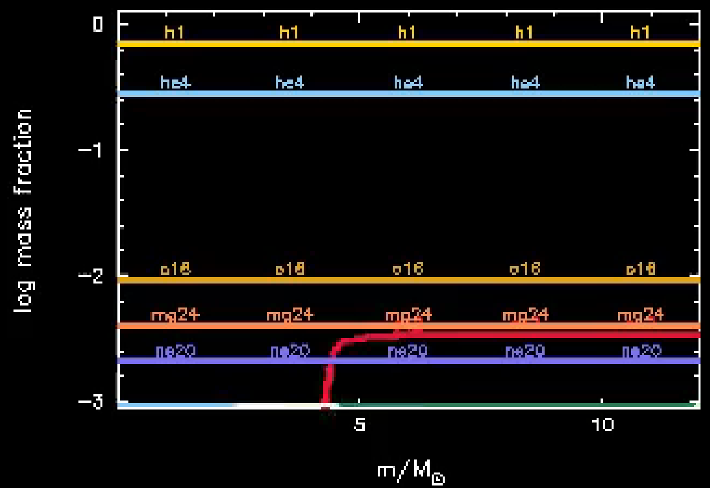
HR



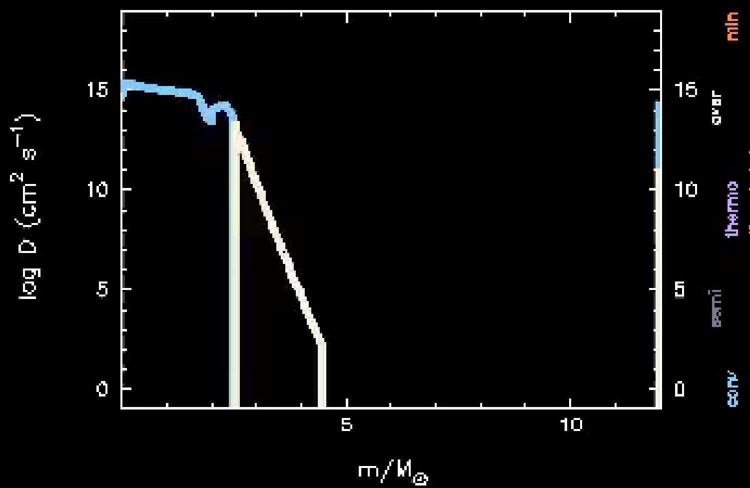
Diagnostic

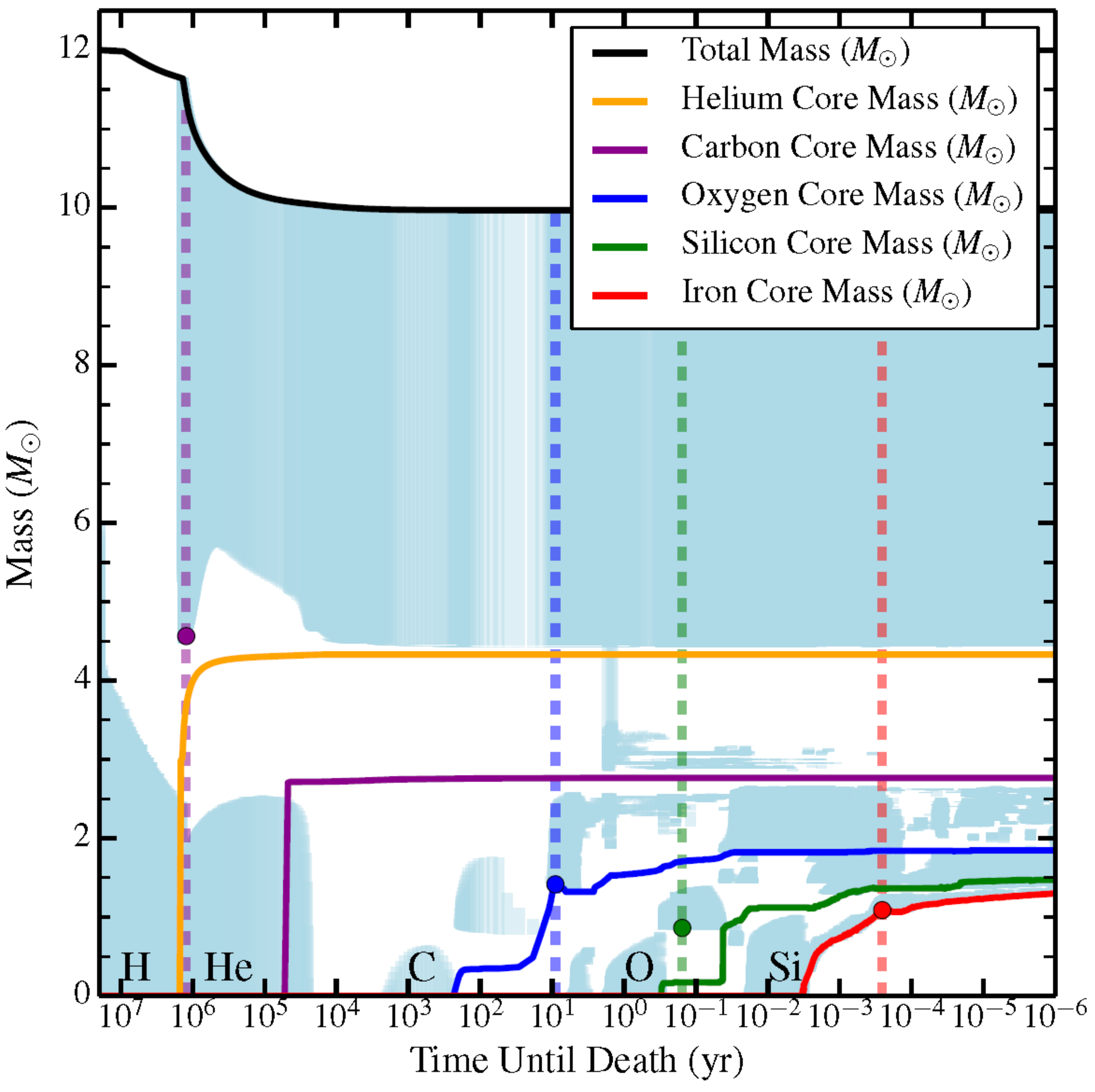


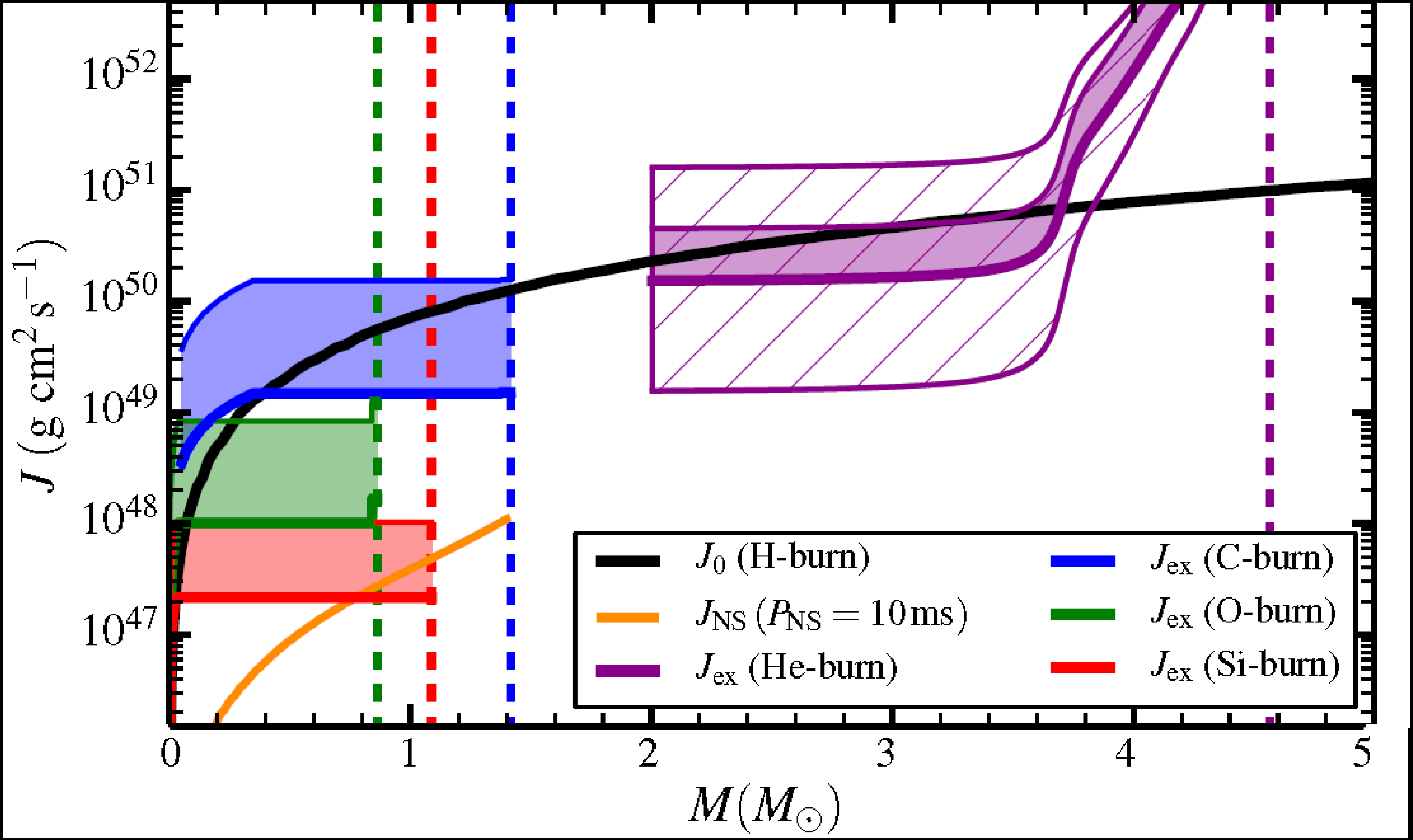
Abundance



Mixing



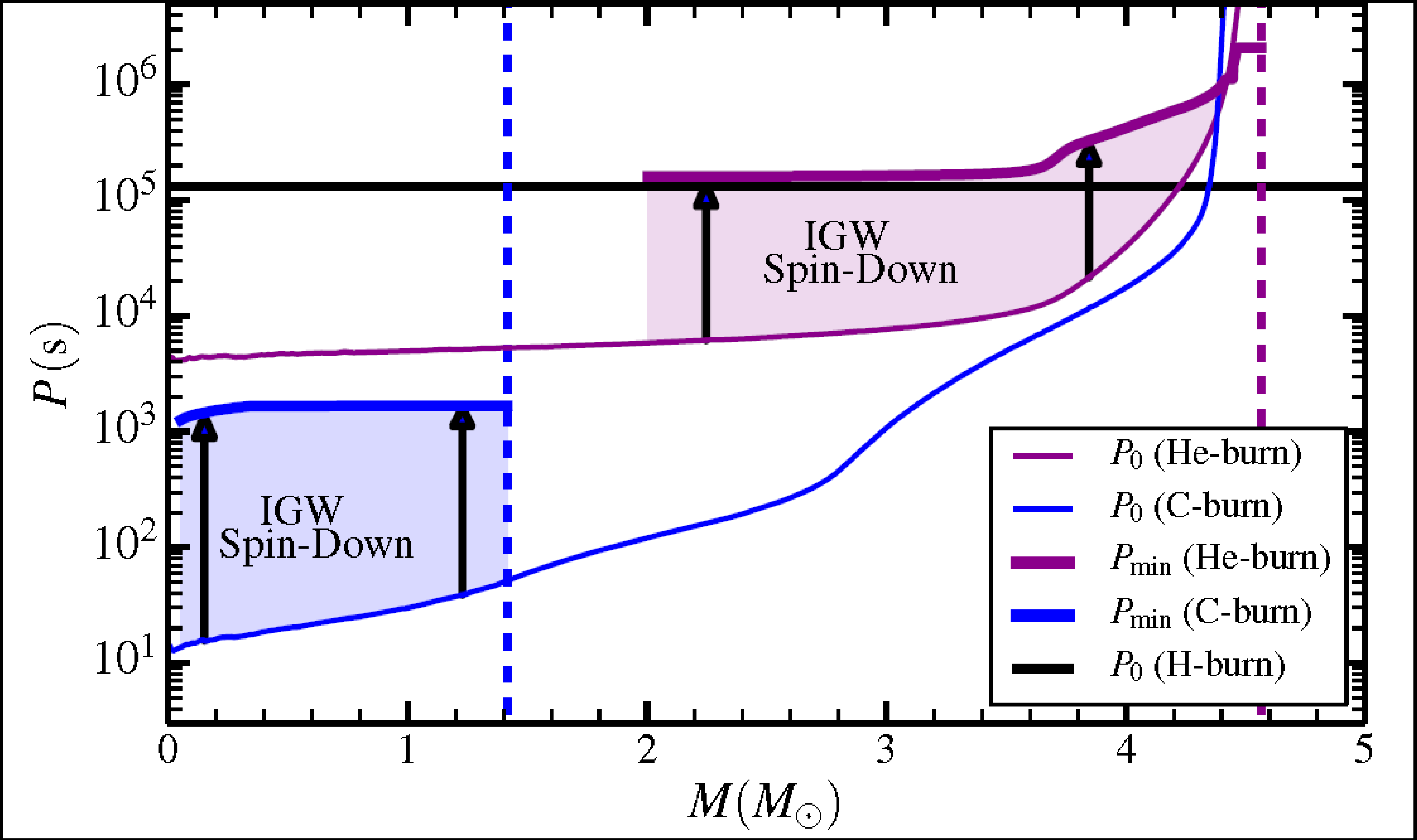

Total Mass ($M_\odot$)
Helium Core Mass ($M_\odot$)
Carbon Core Mass ($M_\odot$)
Oxygen Core Mass ($M_\odot$)
Silicon Core Mass ($M_\odot$)
Iron Core Mass ($M_\odot$)
Mass ($M_\odot$)
0
2
4
6
8
10
12
H
He
C
O
Si
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Time Until Death (yr)


10^{52}
10^{51}
10^{50}
10^{49}
10^{48}
10^{47}
$J\,(\mathrm{g\,cm^2\,s^{-1}})$
0
1
2
3
4
5
$M(M_\odot)$
J_0 (H-burn)
$J_{\rm NS}$ ($P_{\rm NS} = 10\,{\rm ms}$)
$J_{\rm ex}$ (He-burn)
$J_{\rm ex}$ (C-burn)
$J_{\rm ex}$ (O-burn)
$J_{\rm ex}$ (Si-burn)

Wave spin-down

- If wave spin-down time scale is less than shell burning time scale, waves can spin down rapidly rotating core
- Large differential rotation can filter out prograde waves such that only retrograde waves propagate into rapidly rotating regions
- Retrograde waves spin down core until rotation frequency is

$$\Omega_{\rm max} \sim \omega_*$$


10^6
10^5
10^4
10^3
10^2
10^1
$P\,(\mathrm{s})$
0
1
2
3
4
5
$M\,(M_\odot)$
IGW Spin-Down
IGW Spin-Down
P_0 (He-burn)
P_0 (C-burn)
$P_{\rm min}$ (He-burn)
$P_{\rm min}$ (C-burn)
P_0 (H-burn)

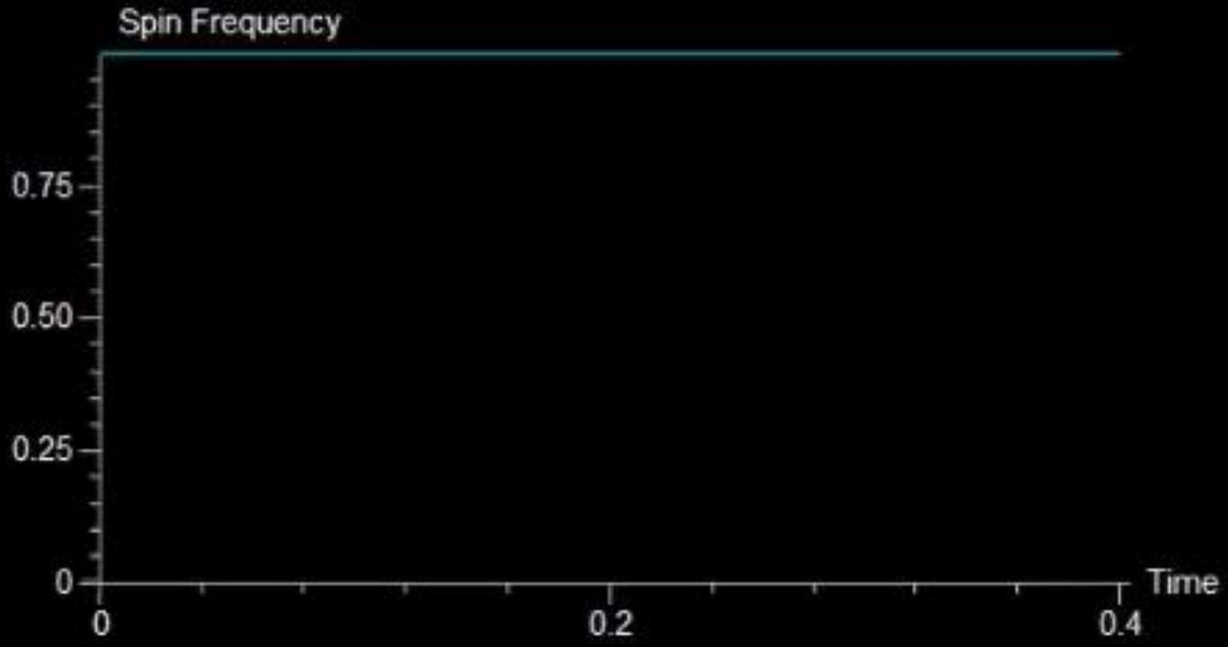
Stochastic core spin-up

- Stochastic convective motions excite series of uncorrelated wave packets which carry AM into the core
- Waves non-linearly break and deposit AM in the core
- Expected AM in core is

$$\sigma_J = \sqrt{\frac{N}{3}} J_w$$

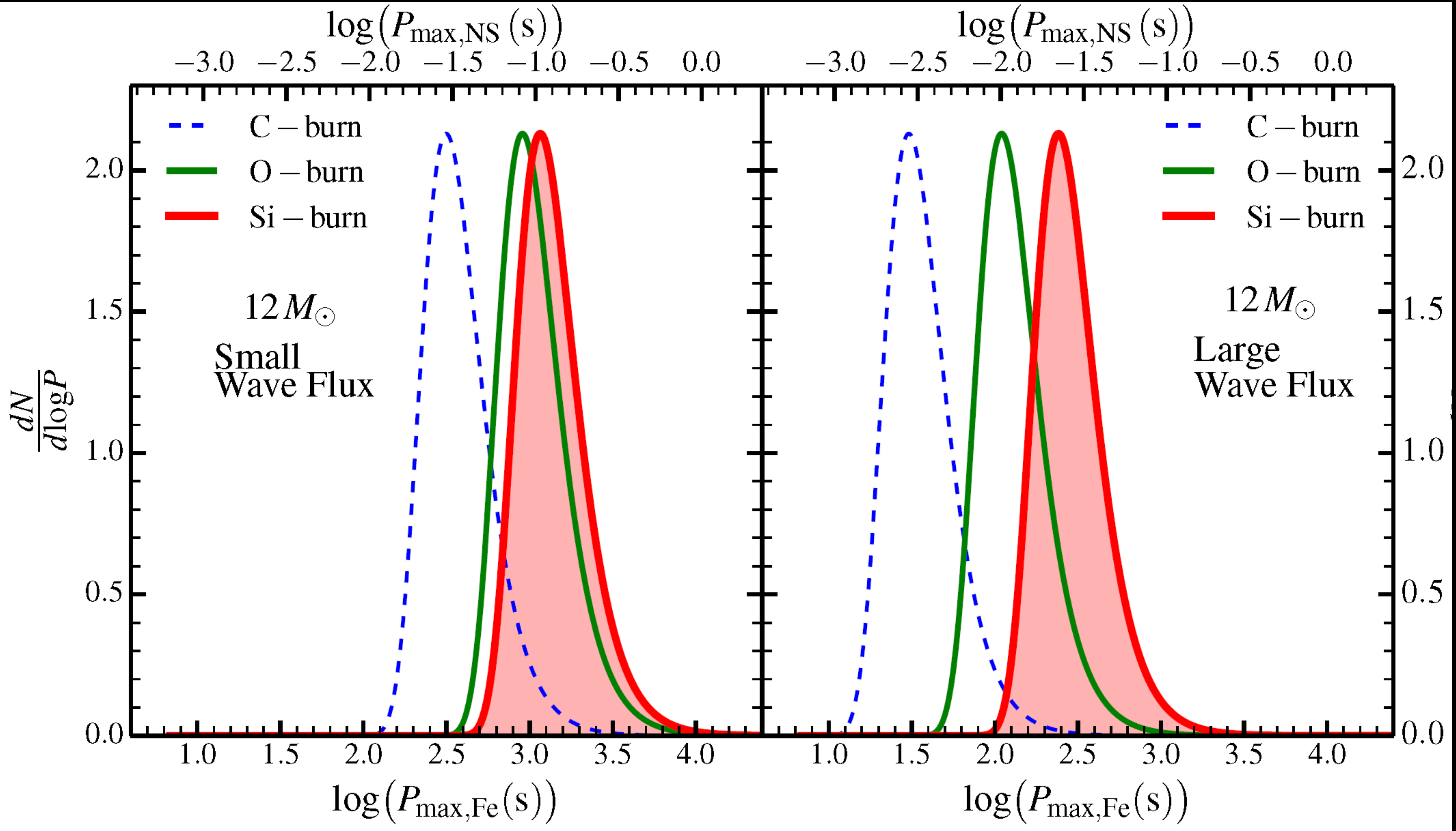
- At end of burning phase, core has expected rotation rate

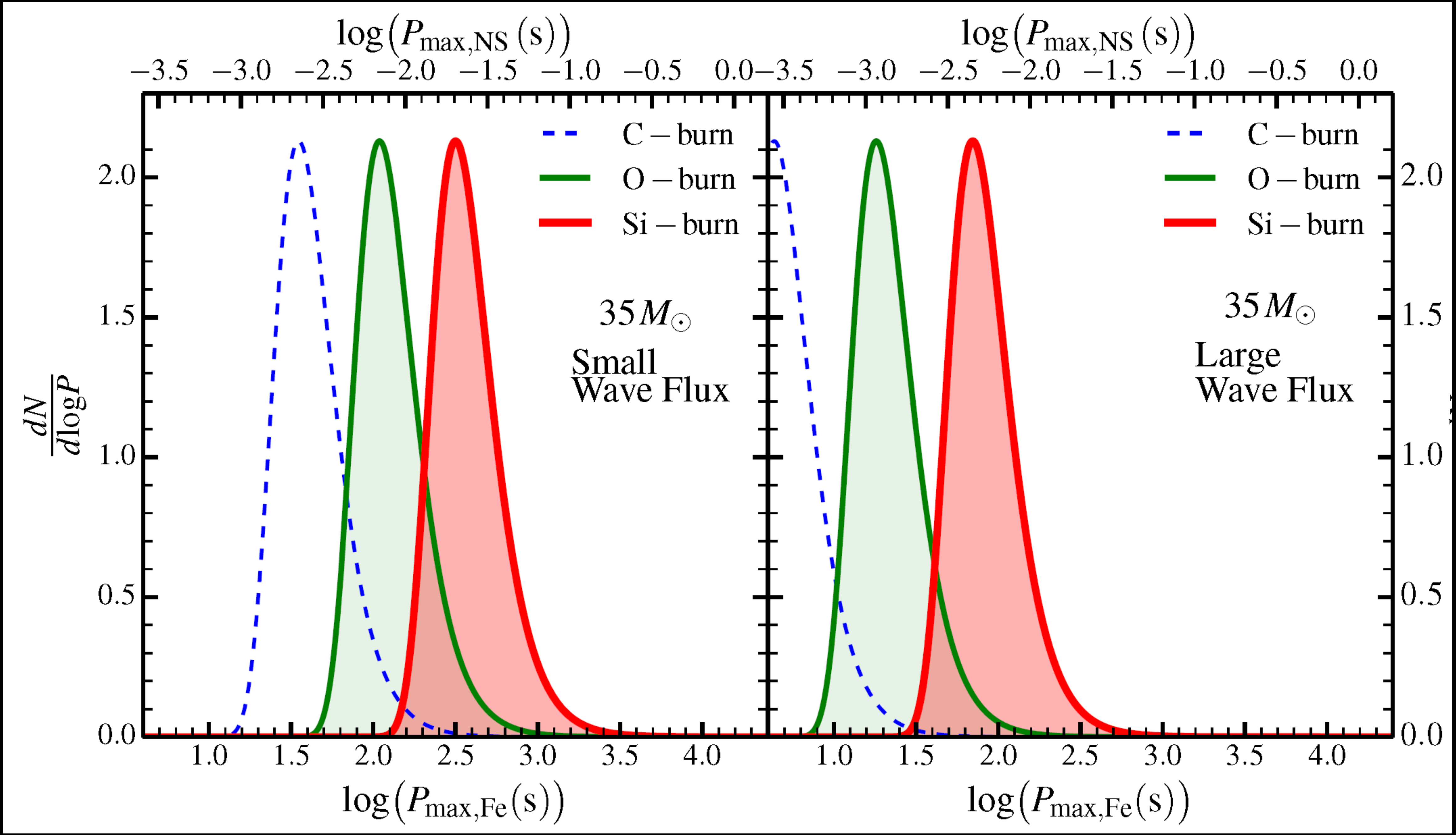
$$\Omega_{\mathrm{ex}} = \sqrt{\frac{4\omega_c T_{\mathrm{shell}}}{3\pi^2}} \frac{J_w}{I_c}$$


Spin Frequency
0.75
0.50
0.25
0
0
0.2
0.4
Time

Caveats

- Stochastic spin-up only important if core was slowly rotating
- Significant amount of differential rotation expected
- Spin-up may be hindered/altered by magnetic torques
    - Magnetic torques unlikely to compete during Si shell burning
- Supernova dynamics may alter spin rate of compact object


log(P_max,NS (s))
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
C − burn
O − burn
Si − burn
12 M☉
Small Wave Flux
dN/dlogP
2.0
1.5
1.0
0.5
0.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
log(P_max,Fe (s))
Large Wave Flux

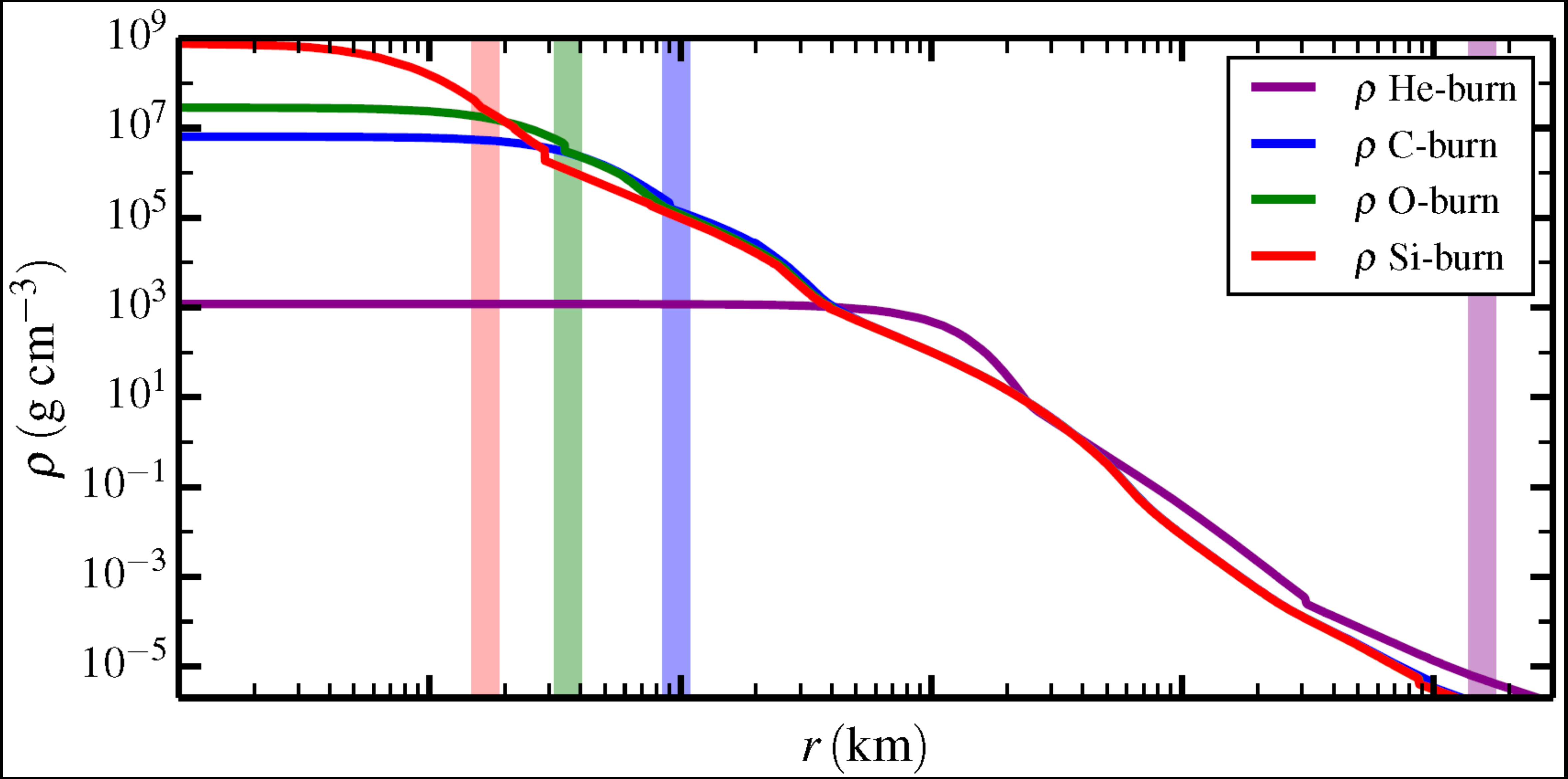
log(P_max,NS(s))
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
dN/dlogP
0.0
0.5
1.0
1.5
2.0
C − burn
O − burn
Si − burn
35 M☉
Small Wave Flux
Large Wave Flux
1.0
1.5
2.0
2.5
3.0
3.5
4.0
log(P_max,Fe(s))

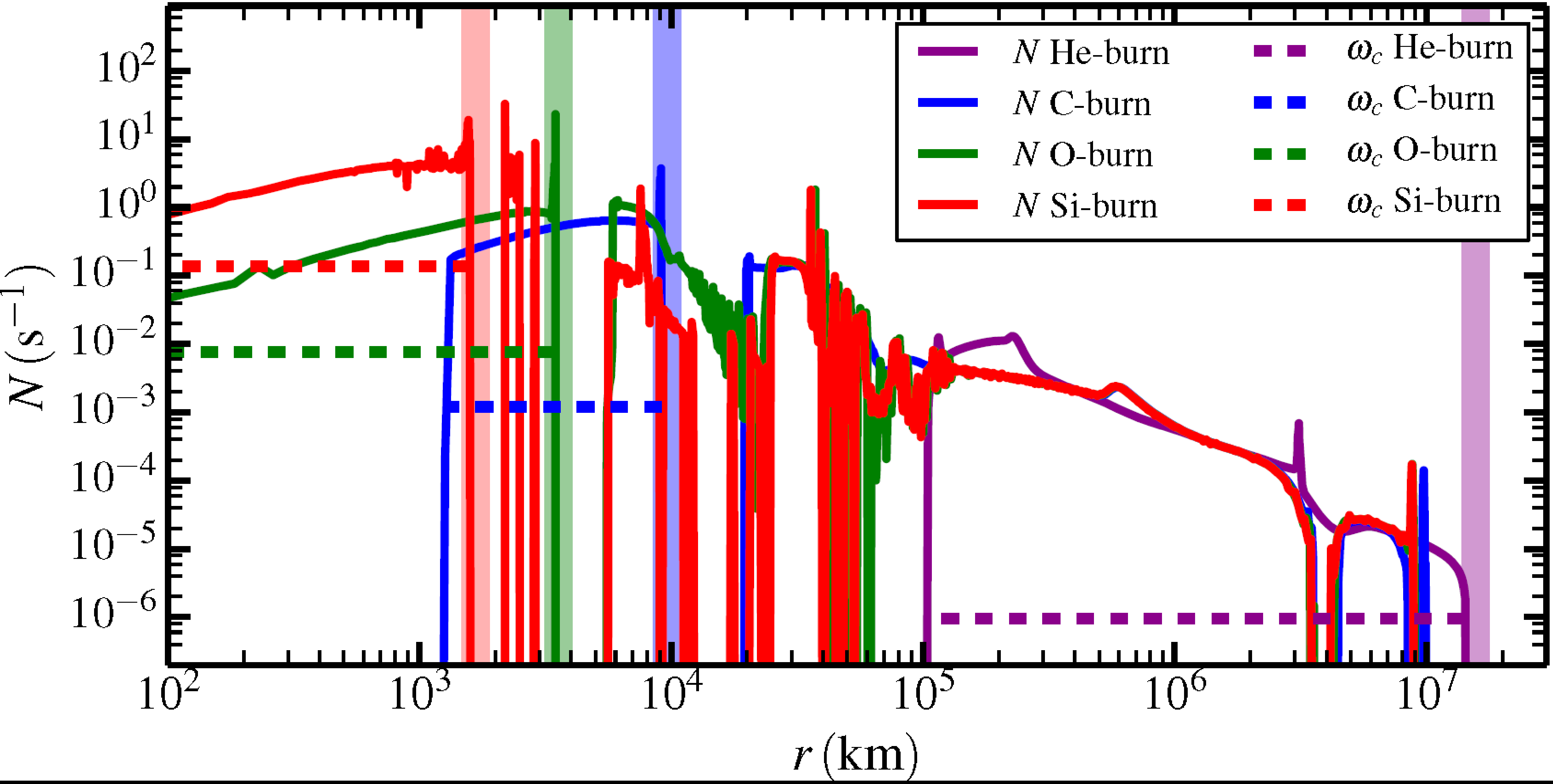
Conclusions

- Internal gravity waves may play crucial roll in pre-collapse dynamics
  - IGW likely set maximum spin rates in cores of red supergiant stars
  - Stochastic spin-up sets minimum spin rate
    - May determine spin rate of many young neutron stars

- Investigate other types of stars/events
  - Black holes, GRB progenitors may have different evolution
  - Low-mass progenitors
    - Off-center shell burning, electron-capture SNe
    - Crab pulsar born rotating somewhat rapidly

Bonus Material




N He-burn
N C-burn
N O-burn
N Si-burn
ω_c He-burn
ω_c C-burn
ω_c O-burn
ω_c Si-burn
$N\,(\mathrm{s}^{-1})$
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$r\,(\mathrm{km})$
10^2
10^3
10^4
10^5
10^6
10^7