Linear Analysis of Shock Dynamics in CCSNe ~Effect of Acoustic Injection and LESA~

Kenichi Sugiura (Waseda Univ.) Collaborators: Kazuya Takahashi (Kyoto Univ.)
Shoichi Yamada (Waseda Univ.)

Introduction and Main Purpose

Successful supernova explosions are supported by multidimensional effects such as

- Standing Accretion Shock Instability (SASI)
- Acoustic mechanism
- Lepton-number Emission Self-sustained Asymmetry (LESA).

Main purpose of this research is investigating the importance of these multidimensional effects qualitatively by using **the linear instability of standing shock.**

Basic Equations

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{v})=0,\quad \frac{\partial}{\partial t}(\rho\boldsymbol{v})+\nabla\cdot(\rho\boldsymbol{v}\boldsymbol{v}+p\boldsymbol{I})=-\rho\frac{GM}{r^2}\frac{\boldsymbol{r}}{r}$$

$$\frac{d\varepsilon}{dt}+p\frac{d}{dt}\left(\frac{1}{\rho}\right)=q,\quad \frac{\partial}{\partial t}(nY_e)+\nabla\cdot(nY_e\,\boldsymbol{v})=\lambda$$

Linearization around spherical steady b.g. with spherical harmonics $Y_{(l,m)}(\theta,\phi)$

+

Laplace transformation $f^*(s):=\int_0^\infty f(t)\mathrm{e}^{-st}dt\quad(s\in\mathbb{C})$

$$M\frac{d\boldsymbol{y}^{*(l,m)}}{dr}(r,s)=\left(sA^{(l)}+B\right)\boldsymbol{y}^{*(l,m)}(r,s)-A^{(l)}\boldsymbol{y}_0^{(l,m)}(r)+\boldsymbol{v}\delta T_\nu^{(l,m)}$$

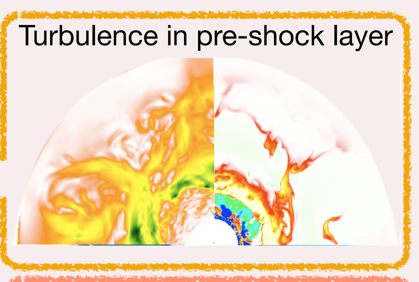
$$\boldsymbol{y}^*(r,s)=\left(\frac{\delta\rho^*}{\delta\rho_0},\frac{\delta v_r^*}{\delta v_{r0}},\frac{\delta v_\perp^*}{\delta v_{r0}},\frac{\delta\varepsilon^*}{\delta\varepsilon_0},\frac{\delta Y_e^*}{\delta Y_{e0}},\frac{\delta v_{\rm rot}^*}{\delta v_{r0}}\right)^T,\ \boldsymbol{v}=\left(0,0,0,\frac{1}{v_{r0}}\frac{\partial q}{\partial T_\nu},\frac{m_b}{\rho_0 Y_{e0}}\frac{\partial\lambda}{\partial T_\nu},0\right)^T$$

Result 1: Effects of acoustic injection

Boundary conditions

Outer b.c. ($r=r_{\rm sh}$): Linearized Rankine-Hugoniot condition

$$\boldsymbol{y}^*(r_{sh},s)=(s\boldsymbol{c}+\boldsymbol{d})\frac{\delta r^*_{sh}}{r_{sh}}(s)+R\boldsymbol{z}^*(r_{sh},s)$$

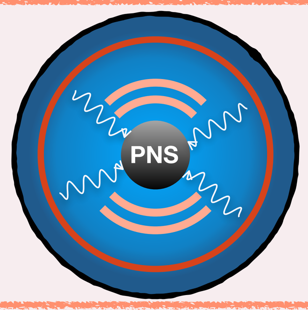


Inner b.c. ($r=r_\nu$):

☑ **Acoustic injection**

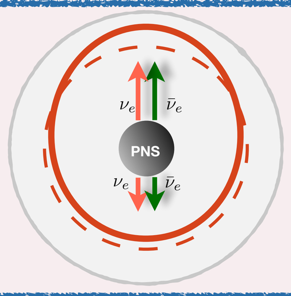
$$\frac{\delta p}{v_{r0}c_s\rho_0}+\frac{\delta v_r}{v_{r0}}=\sin(\omega_{\rm PNS}\,t)$$

Outgoing acoustic mode



☑ **Fluctuations of neutrino temperature**

$$\left(\frac{\partial P}{\partial Y_e}\right)_{\rho,T}\delta Y_e(r_{\nu_e},t)+\left(\frac{\partial P}{\partial T}\right)_{\rho,Y_e}\delta T_\nu(t)=0$$



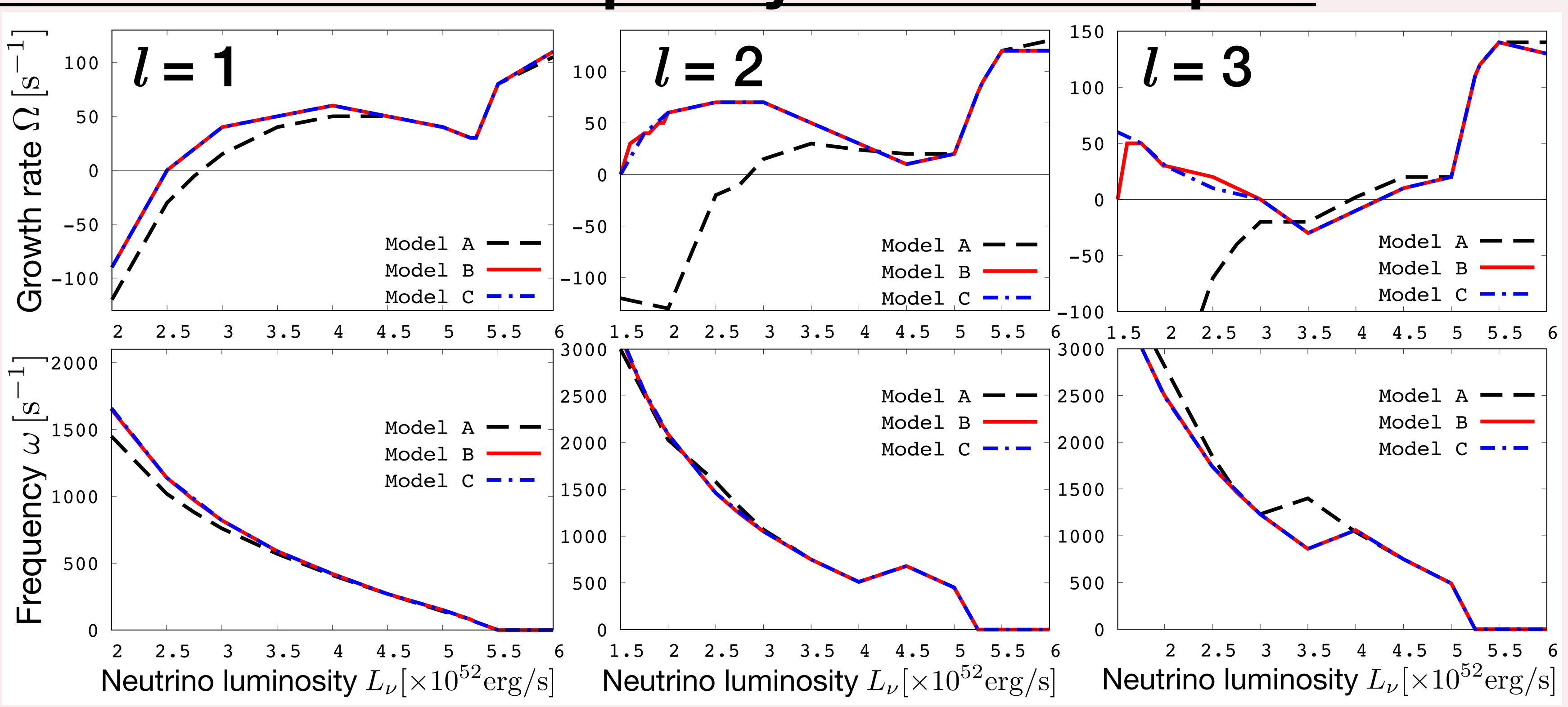
Eigenmodes expansion

$$\frac{\delta r_{sh}}{r_{sh}}(t)=\sum_i a_i\mathrm{e}^{\Omega_i t}\mathrm{e}^{\mathrm{i}\omega_i t+\phi_i}\ \xrightarrow{\text{Laplace trans.}}\ \frac{\delta r^*_{sh}}{r_{sh}}(s)=\sum_i a_i\frac{\mathrm{e}^{\phi_i}}{(s-\Omega_i)-\mathrm{i}\omega_i}$$

Ω_i:growth rate, ω_i:frequency

Ω_i,ω_i emerge as a pole of $\delta r^*_{sh}/r_{sh}$

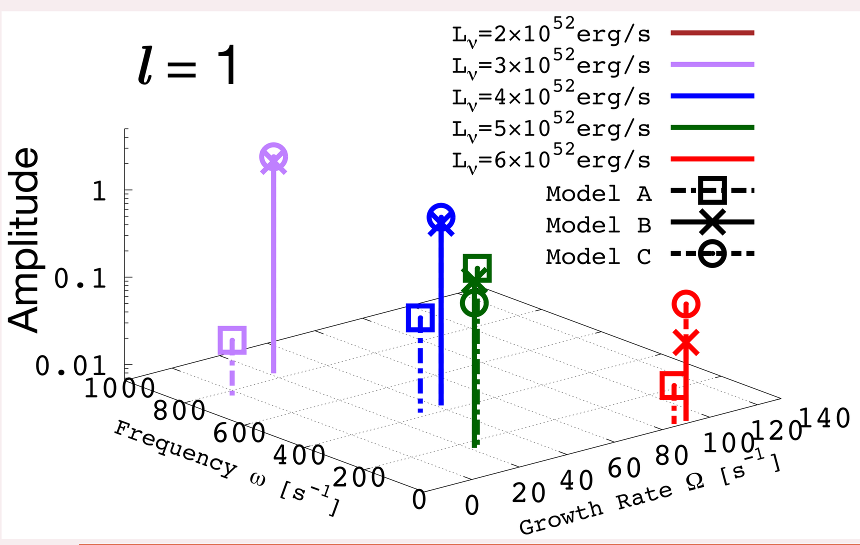
Growth rate and frequency of dominant pole

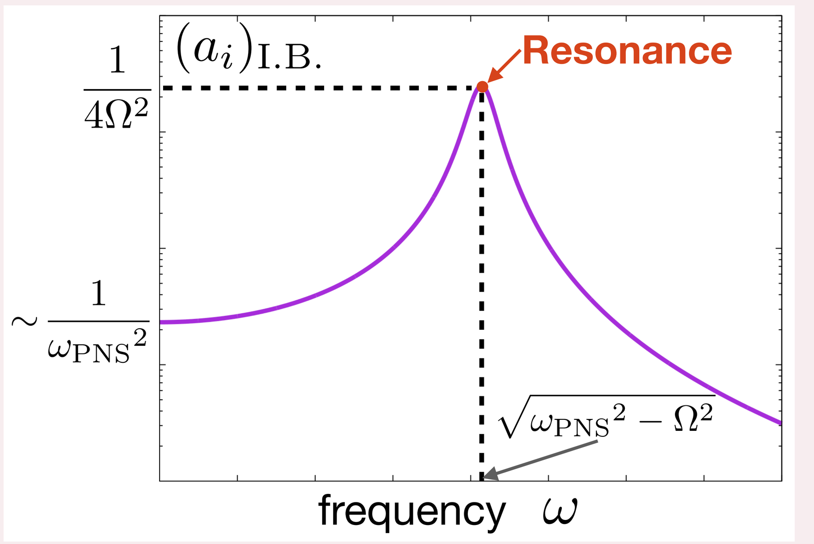


Growth rates of shock deformation are enhanced by acoustic injection.

Amplitude of dominant pole and resonance

$$a_i=(a_i)_{\rm upstream}+(a_i)_{\rm I.B.}\qquad (a_i)_{\rm I.B.}=|(\sin\omega_{\rm PNS}t)^*|=\left|\frac{\omega_{\rm PNS}}{(\Omega+\mathrm{i}\omega)^2+\omega_{\rm PNS}{}^2}\right|$$





→ **Resonance of SASI freq. and g-mode freq. enhance the amplitude**

Result 2: Steady solution of perturbed eq.

LESA is known as systematic deformation of standing shock by asymmetric neutrino emission. But it is unclear which of asymmetry in lepton number flux $F^n_{\nu_e}-F^n_{\bar\nu}$ or deformation of the sum $F_{\nu_e}+F_{\bar\nu}$ is closely related to shock deformation.

Boundary conditions

Outer b.c. ($r=r_{\rm sh}$) :Linearized Rankine-Hugoniot cond.

$$\boldsymbol{y}^*(r_{\rm sh},s)=(s\boldsymbol{c}+\boldsymbol{d})\frac{\delta r^*_{\rm sh}}{r_{\rm sh}}(s)$$

No turbulence in pre-shock layer

Inner b.c. ($r=r_\nu$):

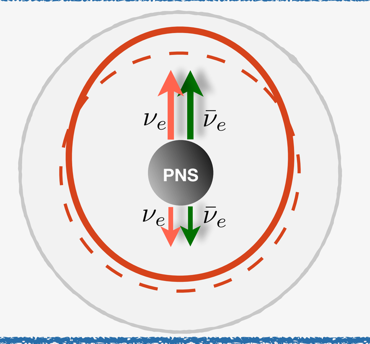
☑ **Fluctuations of neutrino luminosity**

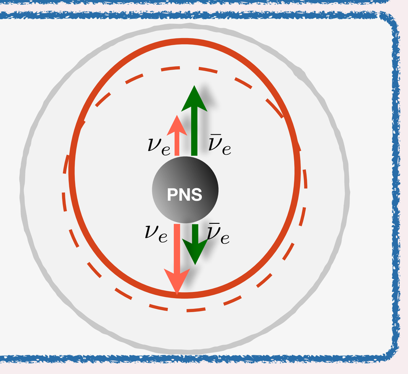
$$\frac{\delta L_{\nu_e}}{L_0}=4\frac{\delta T_{\nu_e}}{T_{\nu_e 0}}+c_{Y_e}\frac{\delta Y_e}{Y_{e0}}$$

$$\frac{\delta L_{\bar\nu_e}}{L_0}=4\frac{\delta T_{\bar\nu_e}}{T_{\bar\nu_e 0}}-c_{Y_e}\frac{\delta Y_e}{Y_{e0}}$$

We consider 2 cases.

$$\begin{cases}c_{Y_e}=0\quad(\Leftrightarrow\delta L_{\nu_e}-\delta L_{\bar\nu_e}=0)\\ c_{Y_e}=3.5\quad(\Leftrightarrow\delta L_{\nu_e}-\delta L_{\bar\nu_e}\neq 0)\end{cases}$$



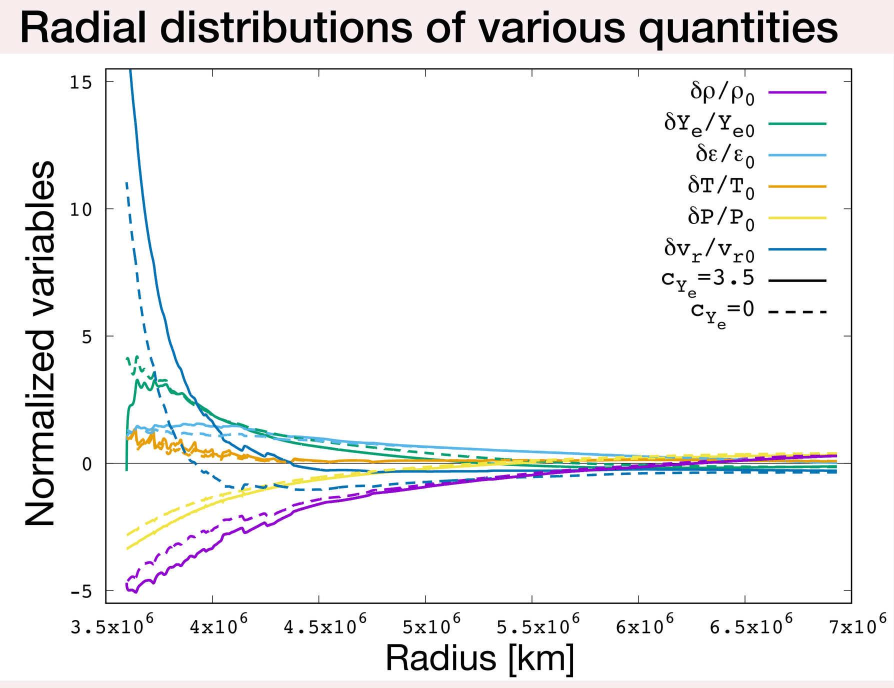


Steady solution

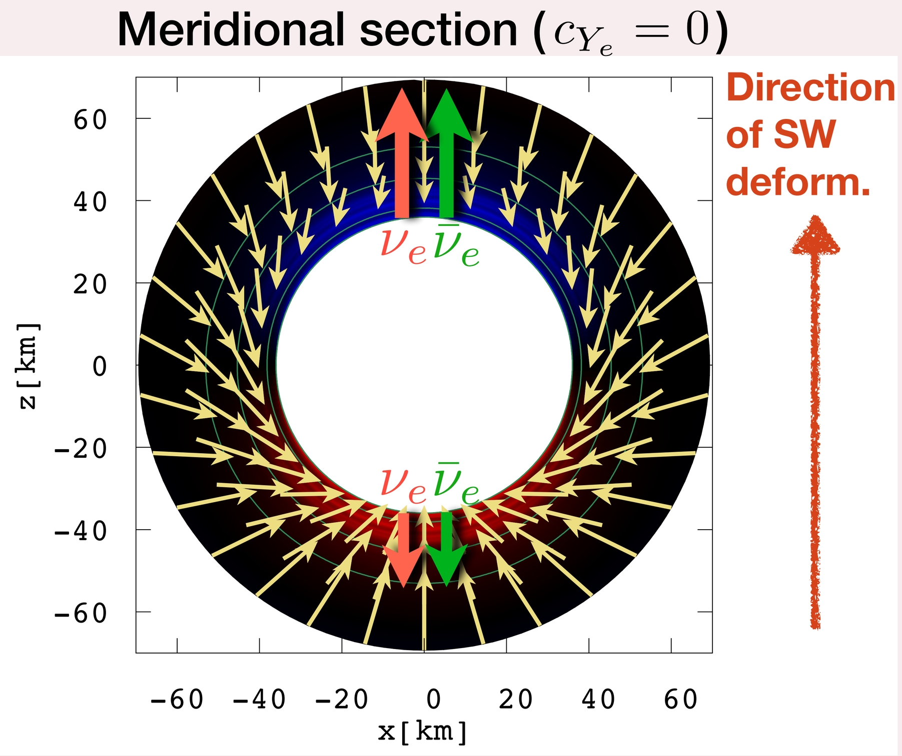
We found self-sustained structure in both cases.

Neutrino luminosity (erg s^{-1})	$c_{Y_e}=3.5$			$c_{Y_e}=0$		
	$\delta r_{\rm sh}/r_{\rm sh0}$	$\delta T_\nu/T_{\nu 0}$	$\delta Y_e/Y_{e0}$	$\delta r_{\rm sh}/r_{\rm sh0}$	$\delta T_\nu/T_{\nu 0}$	$\delta Y_e/Y_{e0}$
2×10^{52}	7.66×10^{-3}	1	-0.650	2.04×10^{-2}	1	0
3×10^{52}	1.51×10^{-1}	1	-0.533	1.86×10^{-1}	1	0
4×10^{52}	4.43×10^{-1}	1	-0.421	4.81×10^{-1}	1	0
5×10^{52}	1.60	1	-0.219	1.63	1	0

Radial distributions of various quantities



Meridional section ($c_{Y_e}=0$)



- Steady structure and sign of shock deformation doesn't change in both cases.
- If LESA exist, shock deformation is suppressed because of stranger cooling.

→ **It is natural to consider the sum of neutrino luminosity fluctuation is closely related with shock deformation.**

5. Summary

☑ Acoustic injection enhances instability of the standing shock especially for lower luminosity.

☑ For lower luminosity, effect of acoustic injection is enhanced because of resonance of SASI and g-mode oscillation of PNS.

☑ Fluctuation of neutrino luminosity does not influence linear instability of standing shock. Futhermore, LESA itself is not closely related with shock deformation.

References

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