

Neutrino Quantum Kinetics in Compact Objects

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Gail McLaughlin

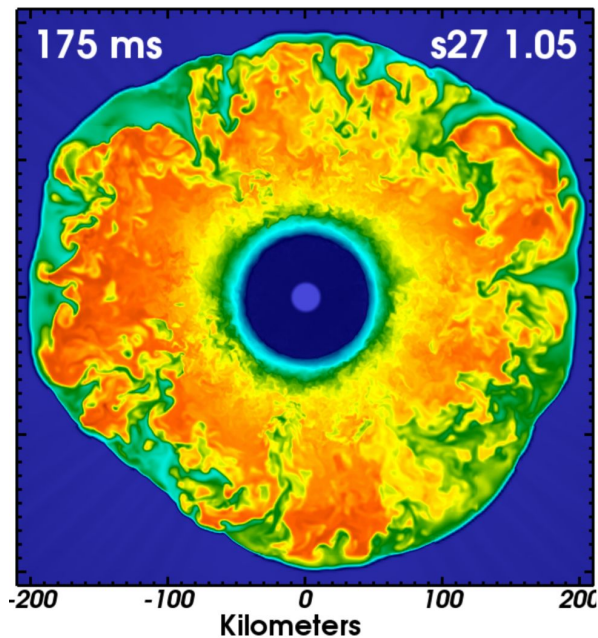
James Kneller

Alexey Vlasenko

Outline

1. Neutrino flavor oscillations in supernova engines
2. **IsotropicSQA**: open-source neutrino quantum kinetics
3. Order-of-magnitude decoherence rates.

Neutrinos in Core-Collapse Supernovae

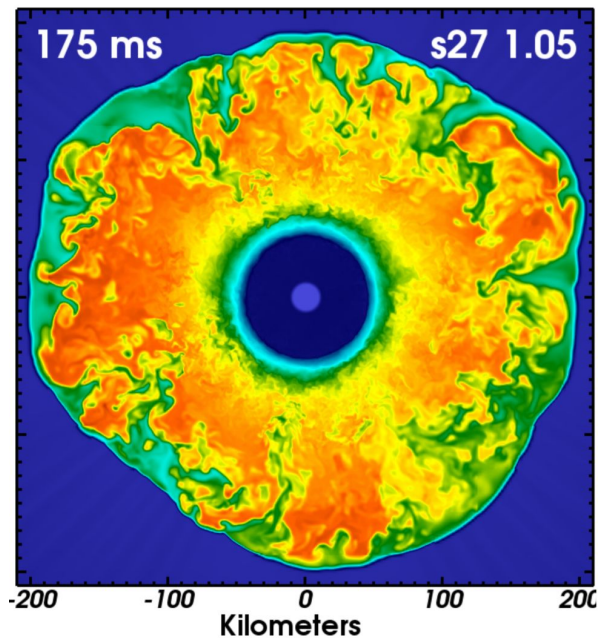


(Couch & O'Connor 2013)

Current Uncertainties

- Progenitor
- Equation of State
- Numerics
 - Resolution
 - Neutrino Transport Method
- Neutrinos
 - Microphysics
 - Flavor oscillations

Neutrinos in Core-Collapse Supernovae



(Couch & O'Connor 2013)

(e.g., Bethe 1985)

Neutrinos drive explosion through **heating**, my drive neutron star **kick**, and may be observed soon.

(e.g., Kusenko 2009)

- Delicate balance determines explodability
- Neutrinos **interact** and **oscillate**

(Historically, never at the same time)

Neutrino Oscillations in Supernova *Engines*

1968 - 2001

Oscillations Proposed and Discovered

Excessively simplified: Pontecorvo (1968), Ahmad+ (2001)

Neutrino Radiation Transport

Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)		
	I	II	III		
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
	u up	c charm	t top	g gluon	H higgs
	d down	s strange	b bottom	γ photon	
	e electron	μ muon	τ tau	Z Z boson	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

LEPTONS (left side)
LEPTONS (bottom left)
GAUGE BOSONS (right side)
VECTOR BOSONS (bottom right)
SCALAR BOSONS (bottom right)

Treat each flavor separately:

$$\frac{d}{d\lambda} \begin{bmatrix} <2.2 \text{ eV}/c^2 \\ 0 \\ \frac{1}{2} \\ \nu_e \\ \text{electron neutrino} \end{bmatrix} = \mathcal{C} \left[\begin{bmatrix} <2.2 \text{ eV}/c^2 \\ 0 \\ \frac{1}{2} \\ \nu_e \\ \text{electron neutrino} \end{bmatrix}, \begin{bmatrix} <0.17 \text{ MeV}/c^2 \\ 0 \\ \frac{1}{2} \\ \nu_\mu \\ \text{muon neutrino} \end{bmatrix}, \begin{bmatrix} <18.2 \text{ MeV}/c^2 \\ 0 \\ \frac{1}{2} \\ \nu_\tau \\ \text{tau neutrino} \end{bmatrix} \right]$$

$$f_{(\nu_e)}(\mathbf{x}, \mathbf{p}, t)$$

Neutrino Quantum Kinetics

Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III	
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	e electron	μ muon	τ tau	Z Z boson
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson

QUARKS (left side, purple and green text)
LEPTONS (left side, green text)
GAUGE BOSONS VECTOR BOSONS (bottom center, red text)
SCALAR BOSONS (right side, yellow text)

But the neutrino flavors are mixed!
 (Pontecorvo 1968, Wolfenstein 1978, Mikheev & Smirnov 1985)

$< 2.2 \text{ eV}/c^2$	$< 2.2 \text{ eV}/c^2$	$< 2.2 \text{ eV}/c^2$
0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
ν_e electron neutrino	ν_μ electron neutrino	ν_τ electron neutrino
$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$
0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tauon neutrino
$< 2.2 \text{ eV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$
0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
ν_e electron neutrino	ν_μ tauon neutrino	ν_τ tau neutrino

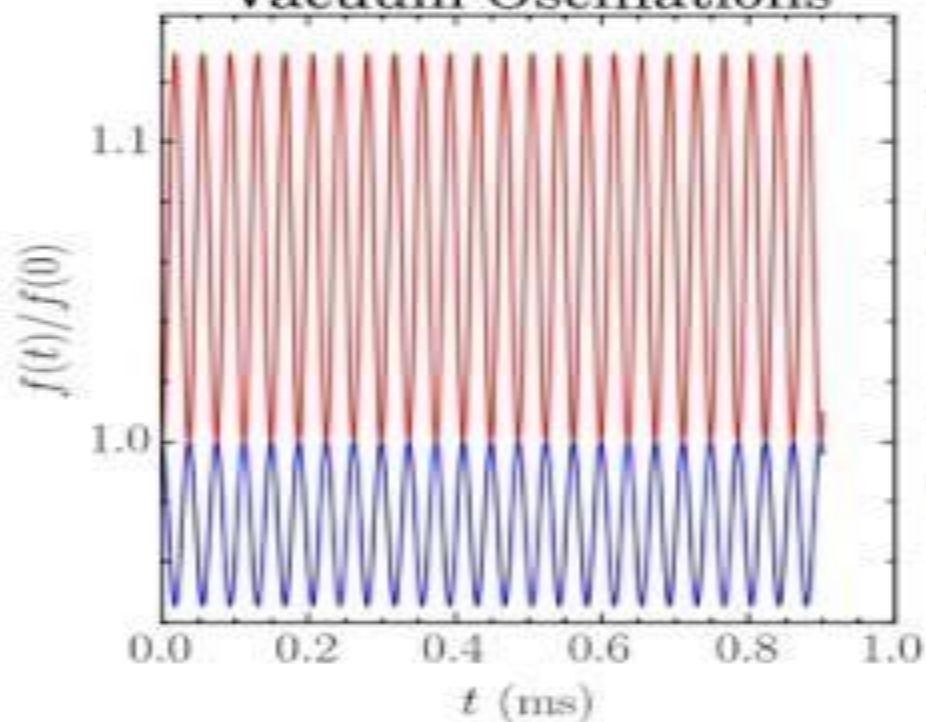
$E = 22 \text{ MeV}$

more ν_e

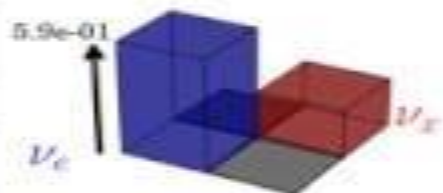


more ν_x

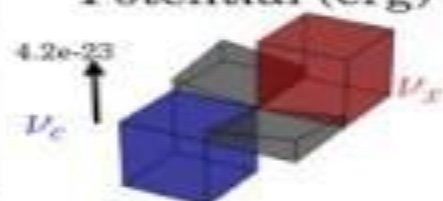
Vacuum Oscillations



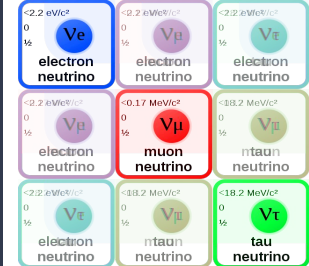
Distribution



Potential (erg)



Flavor Oscillations in Supernova *Engines*



1968 - 2001

Oscillations Proposed and Discovered

Excessively simplified: Pontecorvo (1968), Ahmad+ (2001)

1978 - 2008

NO OSCILLATIONS: In-medium Effects

E.g., Wolfenstein (1978), Mkiheyev & Smirnov (1985), Fuller+ (1992), Quian+ (1993), Fogli+ (2007), Esteban-Pretel+ (2008)

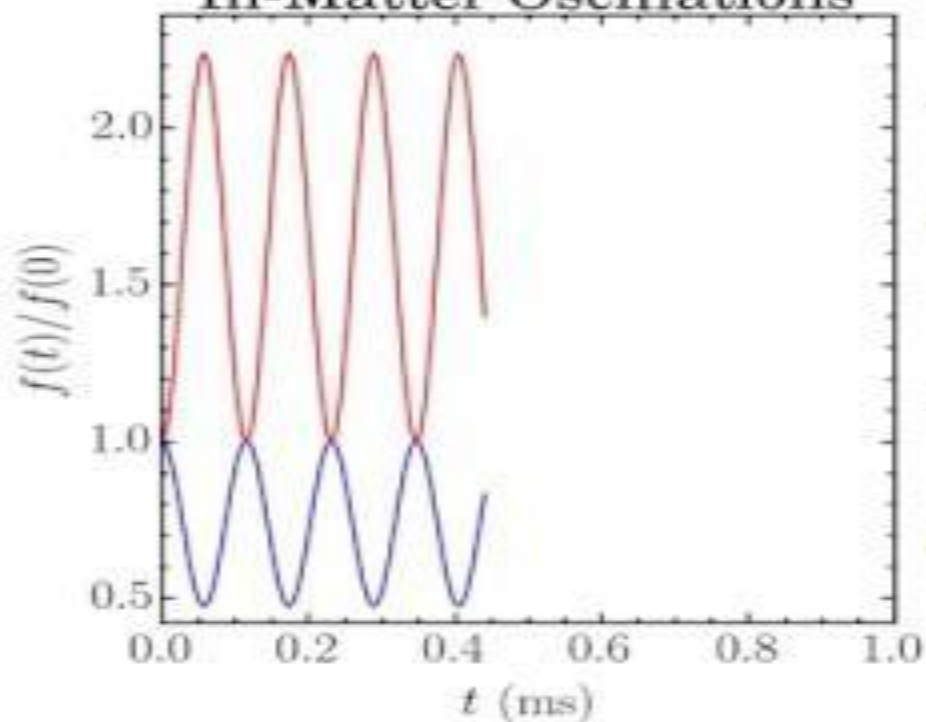
$E = 22 \text{ MeV}$

more ν_e

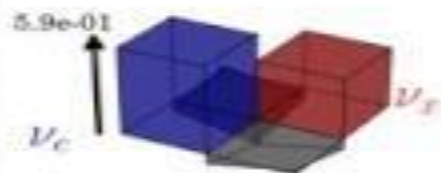


more ν_x

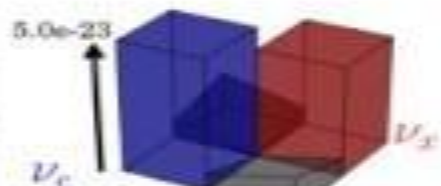
In-Matter Oscillations



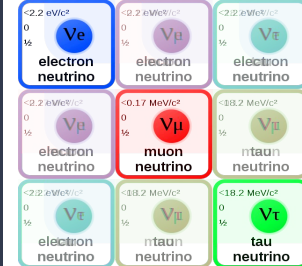
Distribution



Potential (erg)



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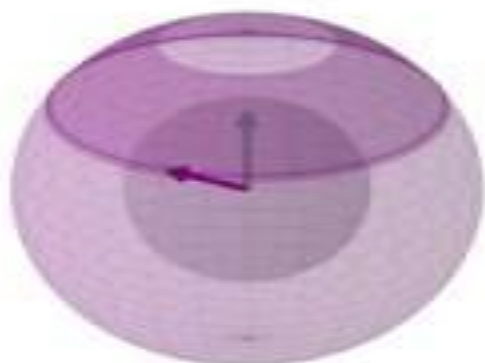
1987 - 2006

OSCILLATIONS: Neutrino-Neutrino Interactions

E.g., Fuller+ (1987,1988), Noetzold & Raffelt (1988), Pantaleone (1992,1998), Pastor+ (2002), Duan+ (2006)

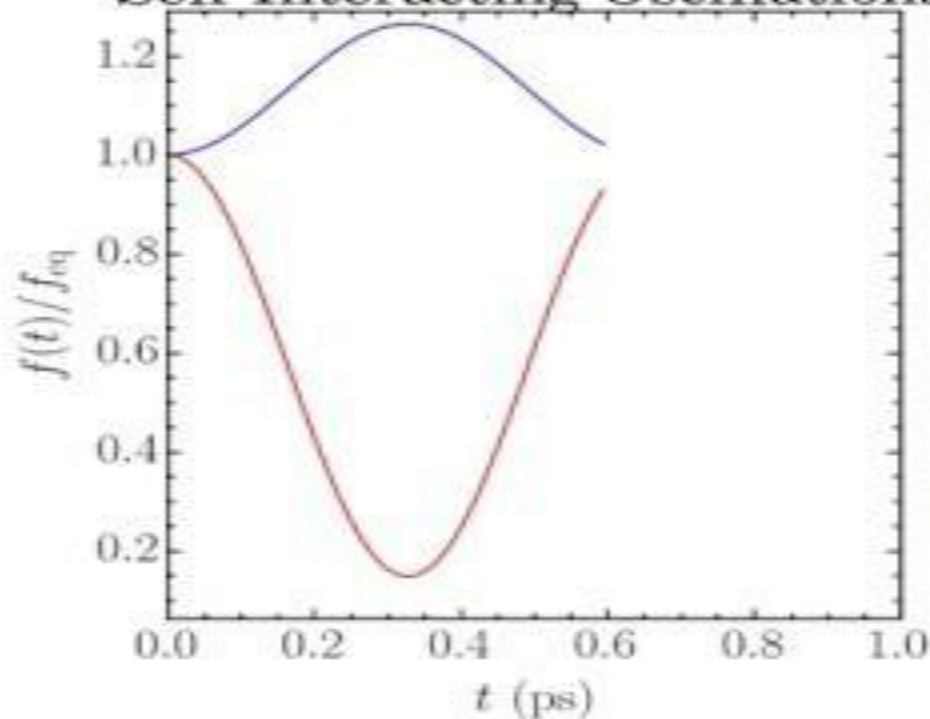
$E = 22 \text{ MeV}$

more ν_e

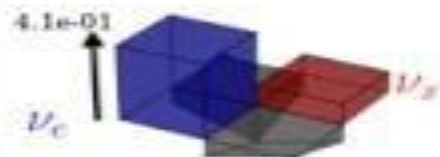


more ν_x

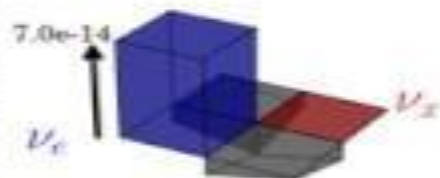
Self-Interacting Oscillations



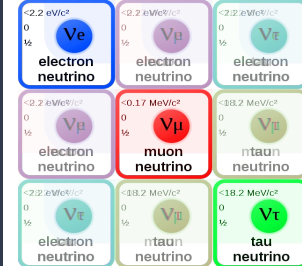
Distribution



Potential (erg)



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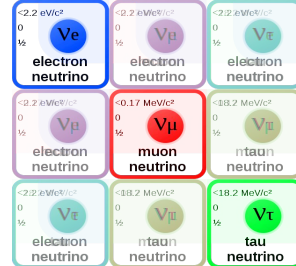
E.g., Fuller+ (1987,1988), Noetzold & Raffelt (1988), Pantaleone (1992,1998), Pastor+ (2002), Duan+ (2006)

2011 - 2012

NO OSCILLATIONS: Direction-Dependent Calculations

E.g., Duan+ (2010), Dasgupta+ (2011), Chakraborty+ (2011)

Flavor Oscillations in Supernova Engines



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Oscillations Proposed and Discovered

Excessively simplified: Pontecorvo (1968), Ahmad+ (2001)

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OSCILLATIONS: Neutrino-Neutrino Interactions

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2011 - 2012

NO OSCILLATIONS: Direction-Dependent Calculations










E.g., Duan+ (2010), Dasgupta+ (2011), Chakraborty+ (2011)

2005 - present

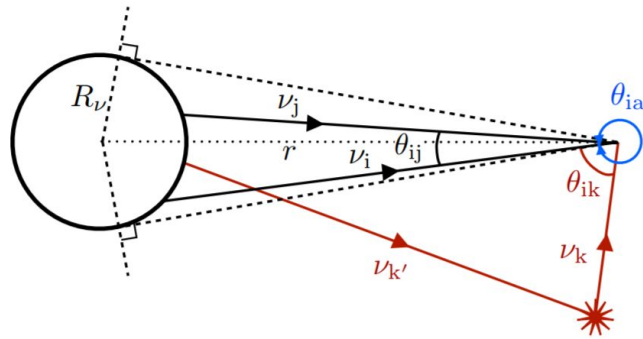
OSCILLATIONS: Direction-Dependent Calculations → New FAST Instability

E.g., Sawyer (2005), Sawyer (2015), Capozzi+(2015), Chakraborty+ (2016), Tamborra+ (2017), Abbar + Volpe (2019), Yi+ (2019), Capozzi+ (2019), Abbar+(2019), Azari+ (2019), Walk+(2019) AND OTHERS

Neutrinos never cease to surprise us!

$\sim 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$  electron neutrino	$\sim 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$  electron neutrino	$\sim 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$  electron neutrino
$\sim 2.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$  electron neutrino	$\sim 0.17 \text{ MeV}/c^2$ 0 $\frac{1}{2}$  muon neutrino	$\sim 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$  tau neutrino
$\sim 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$  electron neutrino	$\sim 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$  tau neutrino	$\sim 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$  tau neutrino

Neutrino Halo Effect

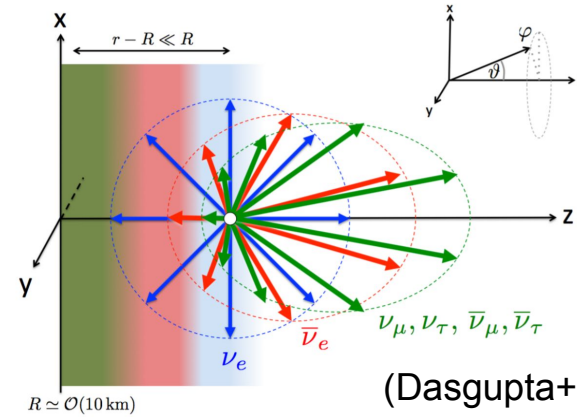


(Cherry+ 2012, 1203.1607)

Small number of reflected neutrinos can change neutrino flavors.

(Also, Cirigliano+ 2018, 1807.07070)

Fast Oscillations

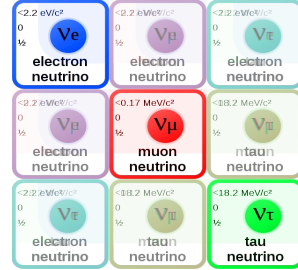


(Dasgupta+ 2017)

Neutrino/anti-neutrino angular dependence cause instabilities

(orig. Sawyer 2005, also Izaguirre+ 2016, Capozzi+ 2017, Dasgupta+ 2018, et al)

Neutrino Quantum Kinetics



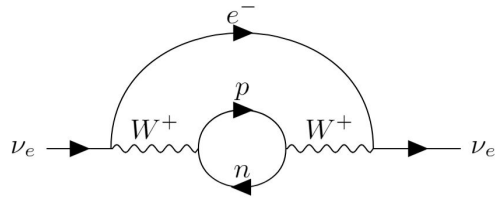
$$\frac{d}{d\lambda} f_{ab}(\mathbf{x}, \mathbf{p}, t) = \mathcal{C} [f_{ab}(\mathbf{x}, \mathbf{p}, t)] - \frac{i}{\hbar c} [\mathcal{H}, f_{ab}(\mathbf{x}, \mathbf{p}, t)]$$

$f_{ab}(\mathbf{x}, \mathbf{p}, t)$
Different flavors experience different potentials.

Cardall 2008
 Vlasenko et al. 2014
 Blaschke+Cirigliano 2016

Well, that's not too bad, right?

Absorption and Emission



1) Read neutrino structure from diagram

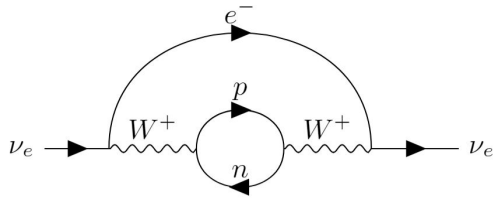
$$C^+ \sim \begin{bmatrix} (1 - f_{ee}) & -f_{e\mu}/2 & -f_{e\tau}/2 \\ -f_{\mu e}/2 & 0 & 0 \\ -f_{\tau e}/2 & 0 & 0 \end{bmatrix}$$

$$C^- \sim \begin{bmatrix} f_{ee} & f_{e\mu}/2 & f_{e\tau}/2 \\ f_{\mu e}/2 & 0 & 0 \\ f_{\tau e}/2 & 0 & 0 \end{bmatrix}.$$

$<2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ ν_e electron neutrino	$<2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ ν_μ electron neutrino	$<2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ ν_τ electron neutrino
$<2.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ ν_μ electron neutrino	$<0.17 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ ν_μ muon neutrino	$<18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ ν_μ muon neutrino
$<2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ ν_e electron neutrino	$<18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ ν_μ tauon neutrino	$<18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ ν_τ tau neutrino

Absorption and Emission

$\sim 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ ν_e electron neutrino	$\sim 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ ν_μ electron neutrino	$\sim 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ ν_τ electron neutrino
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$\sim 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ ν_e electron neutrino	$\sim 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ ν_μ tauon neutrino	$\sim 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ ν_τ tau neutrino



- 1) Read neutrino structure from diagram

$$C^+ \sim \begin{bmatrix} (1 - f_{ee}) & -f_{e\mu}/2 & -f_{e\tau}/2 \\ -f_{\mu e}/2 & 0 & 0 \\ -f_{\tau e}/2 & 0 & 0 \end{bmatrix}$$

$$C^- \sim \begin{bmatrix} f_{ee} & f_{e\mu}/2 & f_{e\tau}/2 \\ f_{\mu e}/2 & 0 & 0 \\ f_{\tau e}/2 & 0 & 0 \end{bmatrix}.$$

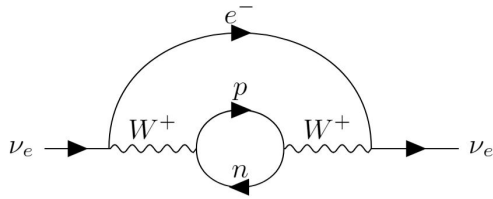
- 2) Require diagonals are the same as **known rates**

$$C_{ee}^+ = j_{(\nu_e)}(1 - f_{ee})$$

$$C_{ee}^- = \kappa_{(\nu_e)} f_{ee},$$

Collision Rates \rightarrow QKE Terms

$\sim 2.2 \text{ eMeV/c}^2$ ν_e electron neutrino	$\sim 2.2 \text{ eMeV/c}^2$ ν_μ electron neutrino	$\sim 2.2 \text{ eMeV/c}^2$ ν_τ electron neutrino
$\sim 2.2 \text{ eMeV/c}^2$ ν_μ electron neutrino	$\sim 0.17 \text{ MeV/c}^2$ ν_μ muon neutrino	$\sim 18.2 \text{ MeV/c}^2$ ν_τ tauon neutrino
$\sim 2.2 \text{ eMeV/c}^2$ ν_e electron neutrino	$\sim 18.2 \text{ MeV/c}^2$ ν_μ tauon neutrino	$\sim 18.2 \text{ MeV/c}^2$ ν_τ tau neutrino



QKE terms
from straightforward
combinations of
ordinary terms

- 1) Read neutrino structure from diagram

$$C^+ \sim \begin{bmatrix} (1 - f_{ee}) & -f_{e\mu}/2 & -f_{e\tau}/2 \\ -f_{\mu e}/2 & 0 & 0 \\ -f_{\tau e}/2 & 0 & 0 \end{bmatrix}$$

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$$C_{ee}^+ = j_{(\nu_e)}(1 - f_{ee})$$

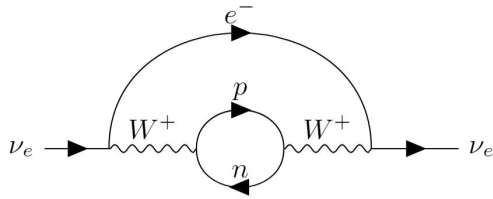
$$C_{ee}^- = \kappa_{(\nu_e)} f_{ee},$$

- 3) Profit.

$$C_{ab} = j_{(\nu_a)} \delta_{ab} - (\langle j \rangle_{ab} + \langle \kappa \rangle_{ab}) f_{ab}$$

Collision Rates \rightarrow QKE Terms

~ 2.2 eMeV/c ² ν_e electron neutrino	~ 2.2 eMeV/c ² ν_μ electron neutrino	~ 2.2 eMeV/c ² ν_τ electron neutrino
~ 2.2 eMeV/c ² ν_e electron neutrino	~ 0.17 MeV/c ² ν_μ muon neutrino	~ 18.2 MeV/c ² ν_τ tauon neutrino
~ 2.2 eMeV/c ² ν_e electron neutrino	~ 18.2 MeV/c ² ν_μ tauon neutrino	~ 18.2 MeV/c ² ν_τ tau neutrino



QKE terms
from straightforward
combinations of
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We also include:

- Electron scattering/annihilation
- Neutrino scattering/annihilation
- Nucleon Scattering
- Effective bremsstrahlung

1) Read neutrino structure from diagram

$$C^+ \sim \begin{bmatrix} (1 - f_{ee}) & -f_{e\mu}/2 & -f_{e\tau}/2 \\ -f_{\mu e}/2 & 0 & 0 \\ -f_{\tau e}/2 & 0 & 0 \end{bmatrix}$$

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2) Require diagonals are the same as **known rates**

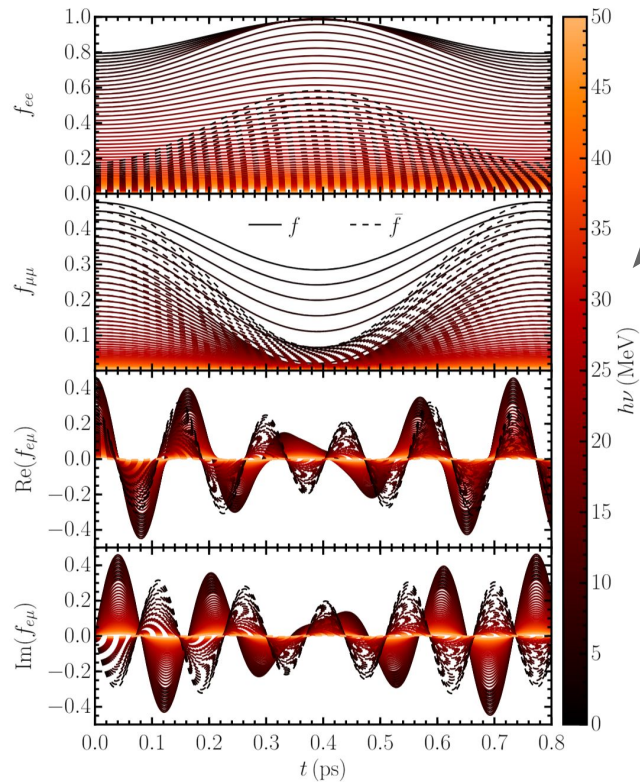
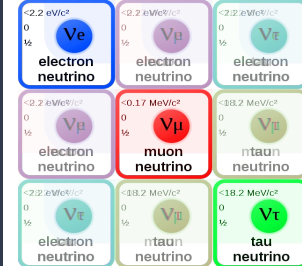
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$$C_{ab} = j_{(\nu_a)} \delta_{ab} - (\langle j \rangle_{ab} + \langle \kappa \rangle_{ab}) f_{ab}$$

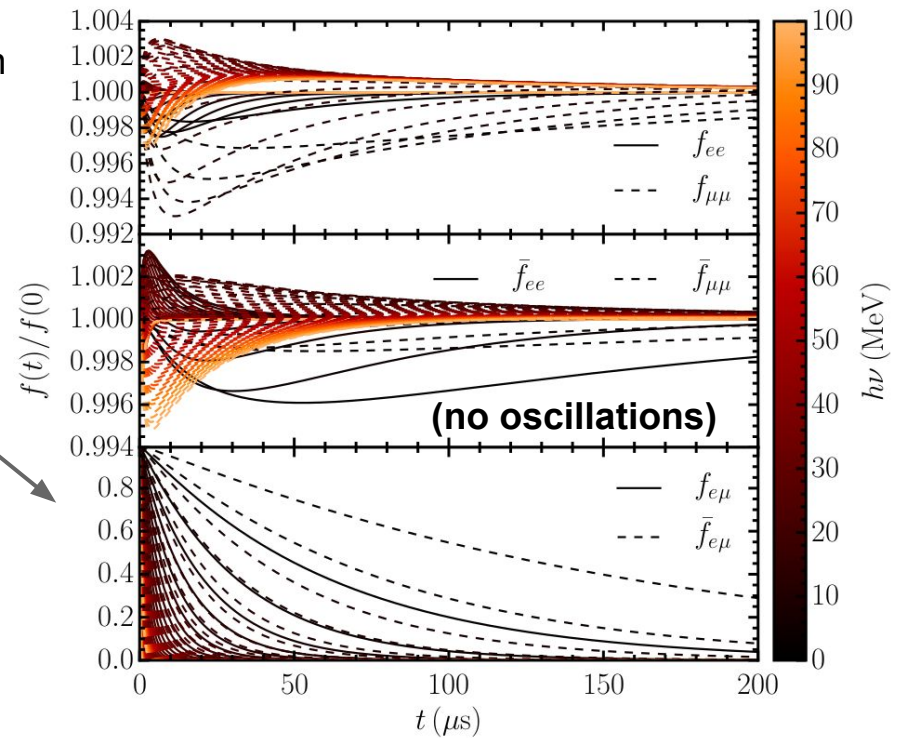
Interactions + Oscillations



Oscillations are on extremely short timescales

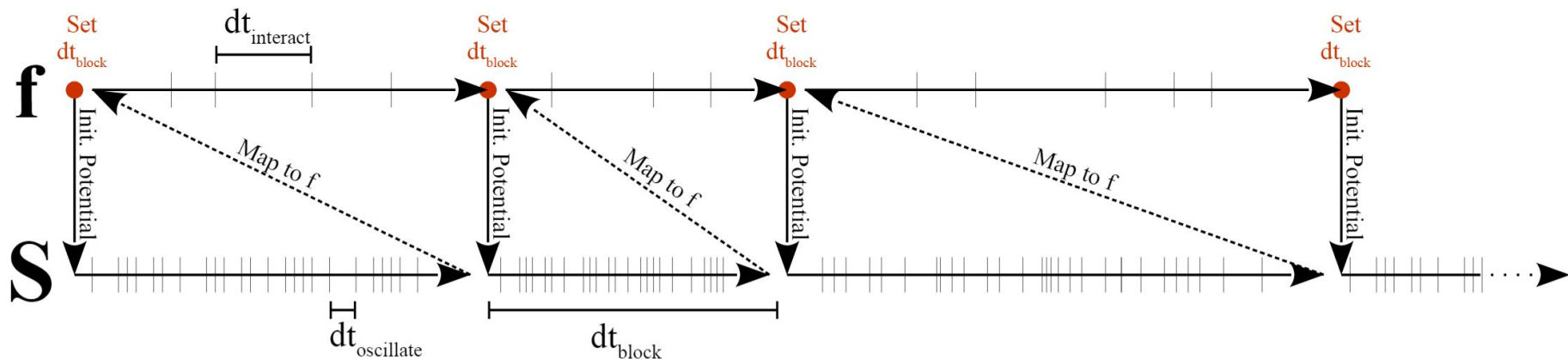
Interactions take 10^7 times longer!

Not possible to evolve directly.



IsotropicSQA

github.com/srichers/IsotropicSQA



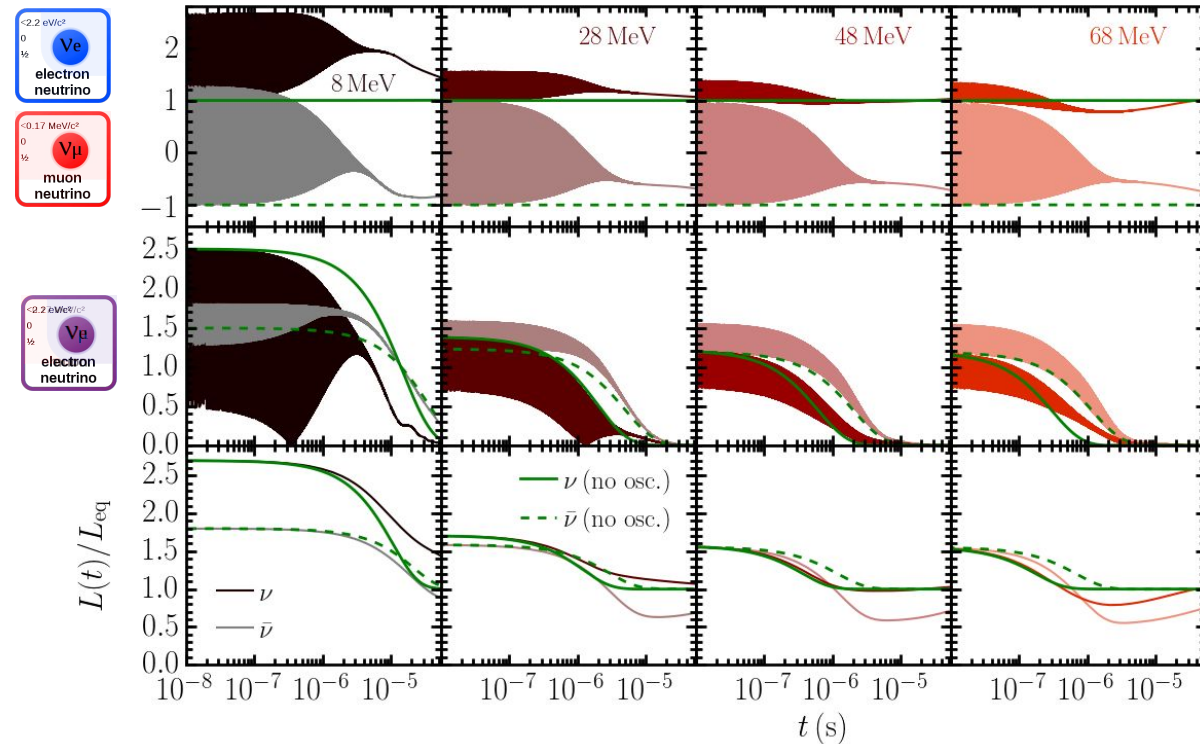
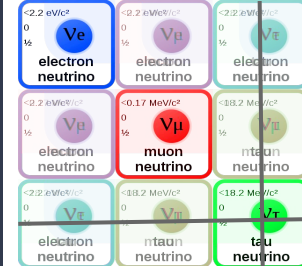
$$\frac{d}{d\lambda} \left[\begin{array}{|c|c|c|c|} \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \end{array} \right] = C \left[\begin{array}{|c|c|c|c|} \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \end{array} \right] - \frac{i}{\hbar c} \left[\mathcal{H}, \begin{array}{|c|c|c|c|} \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \text{blue} & \text{red} & \text{green} & \text{yellow} \\ \hline \end{array} \right]$$

- 1) Evolve **oscillations only** for some “block” time
- 2) Evolve **interactions only** for same amount of “block” time
- 3) Check the impact of the interactions and adjust “block” time

Details:

- Must randomize block time to accurately sample distribution
- Must evolve oscillations with high accuracy (many many timesteps)

Everything Together

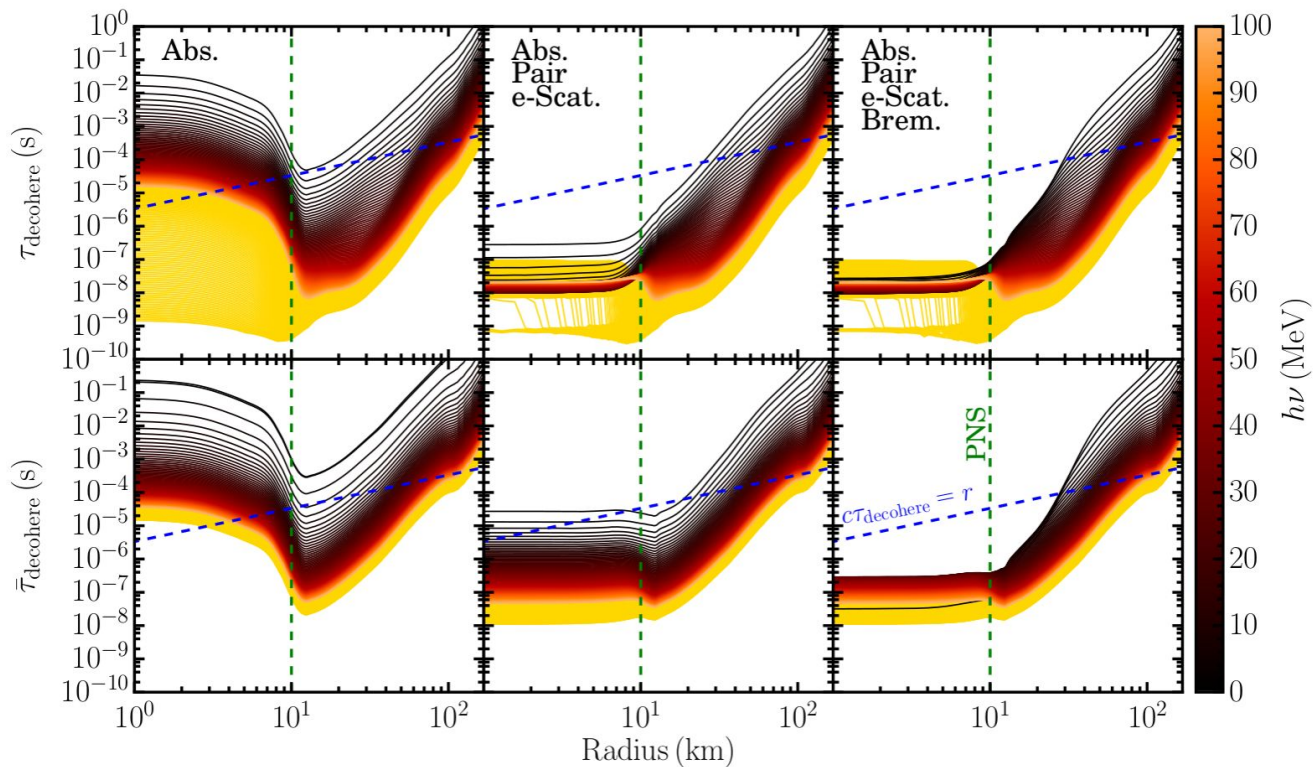
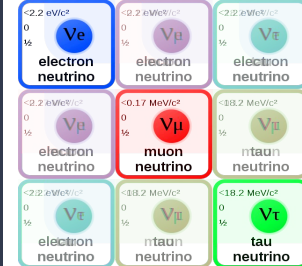


Decoherence with/without oscillations is **similar**

Separate timescales:

- 1) Damping oscillations
- 2) Relaxing to equilibrium

Many collision processes are important!



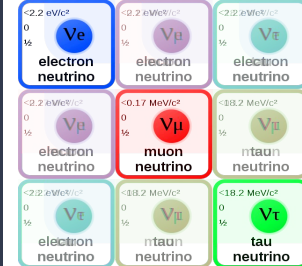
Core:

- Absorption/Emission
- Pair Processes
- Scattering
- Bremsstrahlung
- Neutrino-Neutrino

Heating Region:

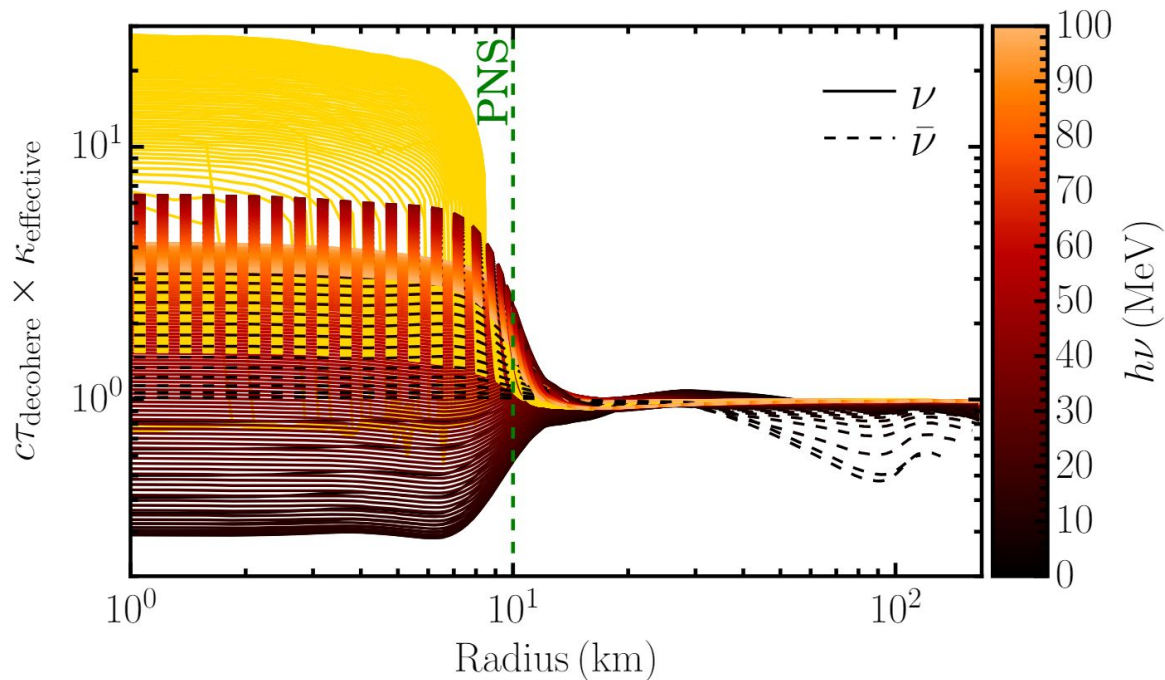
- Absorption/Emission

Simple “effective opacity”

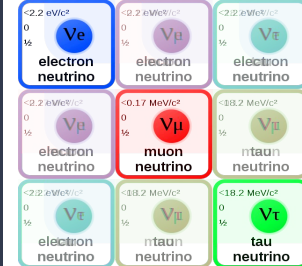


$$\kappa_{\text{effective}} := \kappa_{\text{abs},e\mu} + \frac{1}{2} \tilde{\kappa}_{\text{scat},e\mu}$$

More effective than
mean free path or
absorption opacity



Conclusions



Existing neutrino interaction rates can be extended to **full QKE source terms!**

Effective opacity approximately predicts decoherence rates.

Many collision processes are significant.

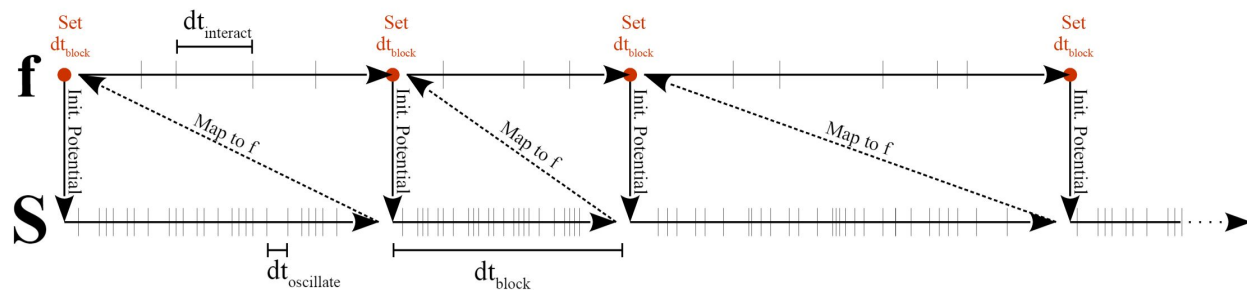
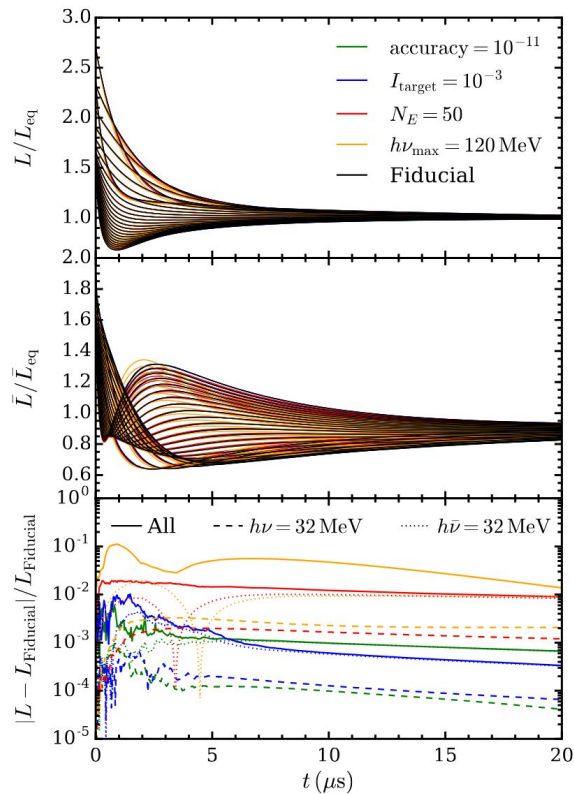
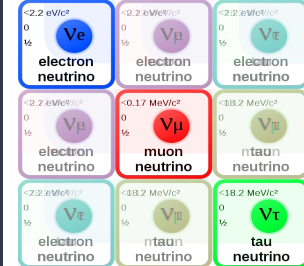
IsotropicSQA simulates **full isotropic QKEs** in supernova conditions.

IsotropicSQA

github.com/srichers/IsotropicSQA

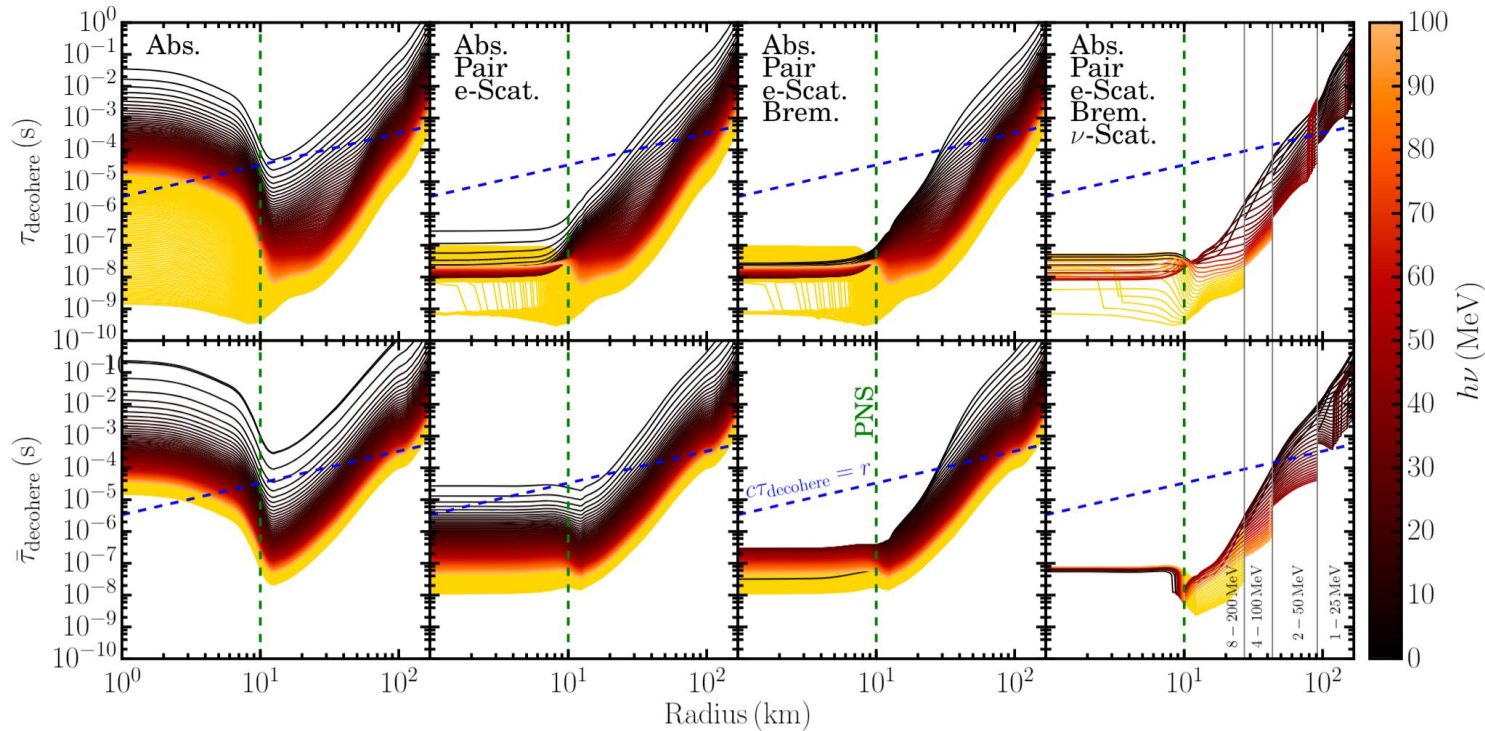
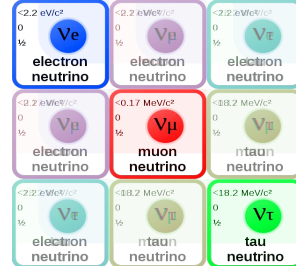
Huge potential for improvement!

Numerical Challenges

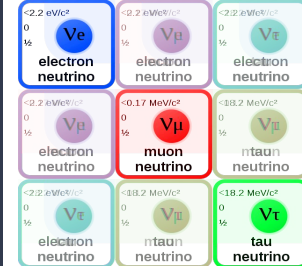


- Few evolution variables are **difficult to parallelize**
- **Conversion between S and f** induces error
- **Neutrino-neutrino interactions** scale with N_E^4 and require evenly-spaced energy grid
- Sensitive to size of the energy domain

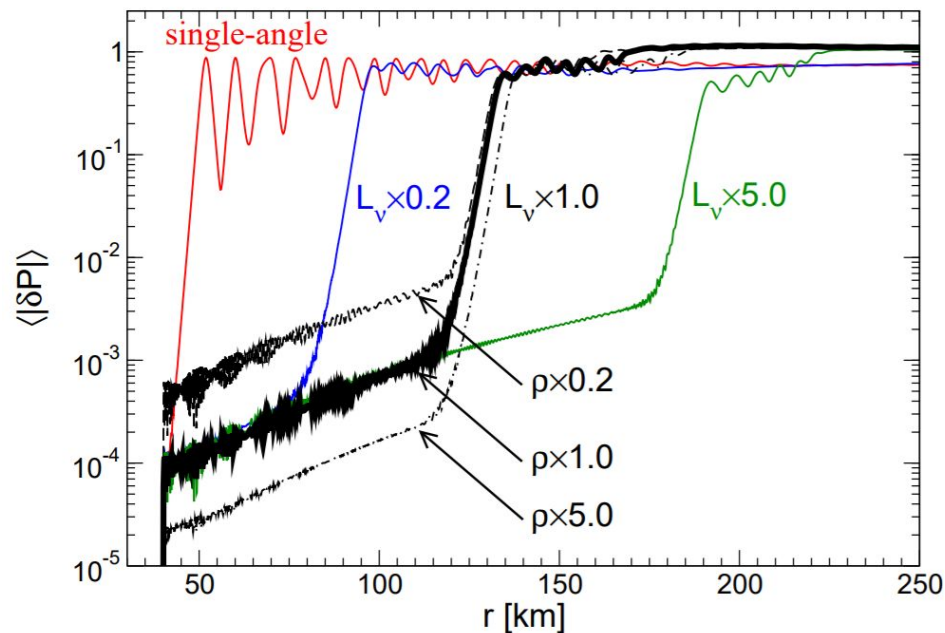
Many collision processes are important!



We don't need to care, right?



- 1) Huge matter potential (Wolfenstein 1979)
- 2) **Multi-angle effects** suppress oscillations (Chakraborty+ 2011, Duan+ 2011, Dasgupta+ 2012)



(Duan+ 2011)